ASHESI UNIVERSITY COLLEGE

DEVELOPING AN OPTIMIZATION MODEL FOR EFFICIENT DISTRIBUTION OF PETROLEUM PRODUCTS. A CASE STUDY OF TEMA OIL REFINERY (TOR)

By

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Declaration

I hereby declare that this dissertation is the result of my own original work and that no part of it has been presented for another degree in this university or elsewhere.

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I hereby declare that the preparation and presentation of the thesis were supervised in accordance with the guidelines on supervision of thesis laid down by Ashesi University College.

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ABSTRACT

This research explores linear programming optimization models for distributing products by tankers in a land transportation system. Tema Oil Refinery (TOR) is used as a case study. The transportation problem is of great economic significance to the Government of Ghana, whose economy is traditionally dominated to a large extent by the oil sector. Any enhancement in the existing transportation procedure has the potential for significant cost savings for the Ghanaian economy. A linear programming model for the TOR problem is constructed in this paper. A set of nodes and arcs is used to form the network, and the decision variables are the different transportation routes. The objective function minimized is formed by summing the products of the decision variables and their corresponding cost coefficients. The constraints of the model include capacity at the depots and the demands of the Oil Marketing Companies (OMCs). The scope of this research covered five (5) OMCs across three (3) capital cities namely; Accra, Kumasi and Takoradi. The resulting linear formulation is greatly simplified and it was solved using Solver (an in-built optimisation tool) in Microsoft EXCEL.

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Chapter I: INTRODUCTION

1.1 Background study

At the core of the development of every modern nation is petroleum (TOR, 2005). Currently, petroleum is among our most important natural resources. We use oil-related products such as gasoline, jet fuel, and diesel fuel to run cars, trucks, aircraft, ships, and other vehicles. The benefits that are gained from using crude oil are numerous and there are not alternatives that can match all the benefits. However, there are ongoing researches into other sources of energy such as wind and solar. This important natural resource should be managed efficiently that is, its processing, distribution and disposal.

Planning and scheduling activities related to product distribution have been receiving growing attention for the past decade. Every company's focal point should be on attending to all its client requirements at the lowest possible cost. As a matter of fact, transportation costs had already surpassed 400 billions dollars in the early eighties (Bodin et al., 1983).

Every decision maker must deal with unforeseen events. He or she must plan for these events as well as respond to them. As a mainstay of transportation analysis, network modelling is an interesting methodology around which to build decision aids for planning unforeseen events. And yet, because unforeseen events in transportation are so diverse, any single methodological approach is best suited only for certain of these decisions (Magnanti, 1983).

Linear Programming (LP) is a mathematical programming technique which involves creating and solving optimization problems with linear objective functions and linear constraints (Ragsdale, 2004). Mathematical Programming is a field of management science that finds the optimal or most efficient way of using limited resources to achieve the objectives of an individual or a business (Ragsdale, 2004). Other MP techniques include Integer Linear Programming, Mixed-Integer Programming and Mixed-Integer Quadratic Programming.

LP has been around since the 1940s and has now reached a very high level of advancement with the dramatic rise in computing power (M.K.Sahdev, K.K.Jain, & Srivastava, 2008). LP has a wide range of practical applications including business, economics, scheduling, agriculture, medicine, natural science, social science, transportation, and even nutrition. This paper is concerned with developing an LP model for Tema Oil Refinery (TOR) that can improve the performance of their existing procedure whereby petroleum products are distributed on a first-come, first-served basis.

1.2 Keywords and Terminology

TOR: Tema Oil Refinery

BOST: Bulk Oil Storage and Transport Company

Bpd: Barrel per day

LP: Linear Programming

GOIL: Ghana Oil Company Limited

OMC: Oil Marketing Company

1.3 Problem Description

In this section, some information about TOR is presented and then a precise statement of the TOR Oil Tanker Scheduling Problem is given.

TOR is the only refinery in Ghana and it is situated at Tema about 24 kilometres from the capital, Accra (TOR, 2005). It is almost identical to three other African refineries; Tanzania (TIPER), Zambia (Indeni) and Congo/Zaire (SOCIR) (Mbendi, 2009). TOR is a refinery with a capacity of 43,000 bpd (barrel per day). It has strategic crude oil storage of 6 x 60,000m^3 (cubic meters) tanks adjacent it, which serve as an important part of the refinery tank farm (TOR, 2005).

Presently, TOR has the authorization to carry on business as refiners and sellers of petroleum. Oil-related products sold by TOR include Liquefied Petroleum Gas (LPG), Diesel, Petrol, Kerosene, Aviation fuel and Fuel oil. Chase Petroleum Ghana Limited and Cirrus Oil Services Limited are privately owned Ghanaian companies who aid in the distribution of oil-related products. Chase Petroleum Ghana Limited is an oil trading and distribution company incorporated in 1999 that trades in crude oil, bulk refined products including gas oil, gasoline, jet oil, LPG (Chase Petroleum Ghana Limited, 2008). Cirrus Oil Services Limited is the only indigenous company with operation license to run bulk storage of petroleum products in the country (Ghana Oil Info).

The Bulk Oil Storage and Transport Company (BOST), a 100% Ghana government owned company is responsible for building facilities to hold strategic stocks. BOST constructed a pipeline and depot system that

included an 80KM pipeline from TOR to Akosombo (at the foot of Lake Volta) with depots at Accra Plains and Mami Water. Other BOST depots are situated at Kumasi in the Ashanti region, Takoradi in the Western Region, and Buipe and Bolgatanga in the northern part of Ghana. A 256km pipeline has also been constructed from Buipe, near the northern end of Lake Volta, to Bolgatanga, near the border shared with Burkina Faso.

The Oil Marketing Companies (OMC) act as liaison officers at all the loading gantries and depots. The liaison officers put in request for products on orders from their Head Offices. TOR's ownership of product and responsibility towards the OMCs both cease after the tanker goes past the refinery or depot gate. Bulk Road Vehicles are the main means of delivering products to the OMC.

To ensure that petroleum products are sold at the same price nationwide, a Unified Petroleum Price Fund (UPPF) has been created where the Government of Ghana pays for the freight of transporting the products to the various depots. The main source of revenue to the TOR is the bulk sale of refined petroleum products to the OMC for distribution to the domestic market. There are currently 38 OMCs licensed to market refined petroleum products in the country (NPA, 2009).

On a typical day, TOR receives orders from the various OMC's and the security agencies (Military and Police Service). The security agencies are given priority over the OMC's hence their orders are met first. Then OMC's

that are in good standing (i.e. those who pay their debt on time) are then served. The rest are served on a first come first served basis. The problem here is to determine how many tankers of gasoline should be transported from a depot to an OMC given the demand of the OMCs, the capacity at the depots and the distribution cost of transporting gasoline from the depots to the OMCs in order to minimise the overall distribution cost.

1.4 Objective and Significance of this Research

This research is intended to highlight the optimum mode for distributing petroleum products from the oil refinery and the various depots to consumers. The main focus of the research is at TOR Oil-Related Products Distribution Problem. This problem is of great economic significance to Ghana especially because of the recent oil find and also most industries and companies need oil-related products to operate. Hence, the efficient distribution of oil-related products would not only minimize cost of distribution for TOR but also the industries and companies would get access to oil-related products when needed.

1.5 Organization of Thesis

This paper will consist of five chapters as follows. Chapter I gives an introduction to the research topic. In Chapter II, literature review relevant to the topic is presented. A linear programming model for the TOR problem is presented in Chapter III. Relevant computational results are given in Chapter IV. A summary of this research is given in Chapter V.

Chapter II: LITERATURE REVIEW

2.1 Introduction

This chapter explores literature relevant to the topic. A linear programming problem is a problem in which the goal to find the maximum or minimum value of a linear expression

ax + by + cz + . . . (the objective function)

subject to a number of linear constraints of the form

ax + by + cz + . . .≤ n

   or

ax + by + cz + . . .≥ n

The review is divided into two categories:

(1) Network flow problems and

(2) Relevant mathematical programming model problems.

Category (2) is briefly touched upon and it is not intended to provide a comprehensive review of land transportation problems. They are presented to provide some insights on some of the existing techniques and algorithms employed to approach relevant land transportation problems.

2. 2 Network Flow Problems

Network flow problems are linear programming problems in which the objective is to minimize the cost, penalty or distance from a source to a destination. These problems share a common characteristic – they can be described or displayed in a graphical form called a network. The network consists of nodes connected by arcs, each arc having a given direction and a limited capacity.

Hsu and Cheng (2002) developed a generalized network flow optimization model for long-term supply-demand analysis for basin-wide water resources planning. A set of nodes and arcs were used to form the network, and the decision variables were reservoir storage and water supply for public and agricultural uses. The objective function to be minimized was formed by summing the products of the decision variables and their corresponding cost coefficients. The constraints of the model included continuity equations, reservoir operation rule curves, reduced water supply due to water shortage, and evaporation losses from reservoirs. The formulated network model was solved by an efficient embedded generalized network solver (EMNET). The developed model was applied to a river basin located in the northern part of Taiwan. The developed model was then used to analyze future water supply-demand conditions for the area.

Munkres (1957) presented algorithms for the solution of the general assignment and transportation problems. In this paper, a statement of the algorithm for the assignment problem appears, along with a proof for the

correctness of the algorithm. The remarks which constitute the proof are incorporated in passing into the statement of the algorithm. The algorithm was then generalized to one transportation problem.

Ford Jr. et al. (1956) gave a simplified description of a new computing procedure for the Hitchcock- Koopmans transportation problem (the case study used), together with a step by step solution of an illustrative example. The procedure was based on Kuhn's combinatorial algorithm for the assignment problem and a simple "labelling process" for solving maximal flow problems in networks. The proposed computation appeared to be considerably more efficient than the specialized form of the simplex method which is in common use.

Cooper (1972) defined a problem type, called the transportation-location problem that could be considered a generalization of the Hitchcock - Koopmans transportation problem in which, in addition to seeking the amounts to be shipped from origins to destinations, the optimal locations of these sources with respect to a fixed and known set of destinations were also found concurrently. This new problem was characterized mathematically, and exact and approximate methods were presented for its solution.

White (1972) studied the class of dynamic transhipment problems. These are transportation problems that are characterized by the movement of vehicles and goods from location to location over time. Such movements can be represented by a network. The author states that if no directed

cycles exist in this network, then an inductive algorithm can be used to optimize the flow of a homogeneous commodity for a linear cost function. This algorithm could be modified to handle networks in which there are directed cycles.

Yan (1988) presented a heuristic method for scheduling of trucks from many warehouses to many delivery points subject to constraints on truck capacity, travelling time, and loading and unloading time. He considered the truck scheduling problem faced by STARLINK, a warehousing and distribution Company based in Hong Kong. This heuristic method was used to build a complete schedule. The author reported the success of this method for the STARLINK Company with 8.8% average cost improvement over the previously used manual method.

Charnes and Cooper (1954) explored transportation-type problems and models in detail. Transportation-type problems have certain features which make it possible to devise special computational techniques which are extremely simple to understand and apply. An illustrative example was presented in this paper which explained the "stepping stone" method for solving these kinds of problems.

Bowman (1956) states that; with fluctuating sales, a manufacturer must have fluctuating production, or fluctuating inventory, or both. Penalties are associated with either type of fluctuation. He suggested that the problem may be placed into a transportation-method framework. Furthermore, he made it known that many transportation problems may be extended to include multiple time periods where it was meaningful.

Kataoka (1963) proposed a stochastic programming model which considers the distribution of an objective function and probabilistic constraints. He derived a nonlinear programming problem with linear inequalities constraints by applying the model to a transportation type problem, and showed that it could be solved by iteration of linear programming.

2.3 Relevant Mathematical Programming Model Problems

The following is a brief survey on scheduling problems solved by other mathematical programming techniques not necessarily linear programming. This survey is intended to provide insights into some of the existing techniques and algorithms employed to tackle these problems.

Lee et al. (1996) addressed the problem of inventory management of a refinery that imports several types of crude oil which are delivered by different vessels. A mixed-integer optimization model was developed which relies on time discrimination. The formulation and solution method was applied to an industrial-size problem involving 3 vessels, 6 storage tanks, 4 charging tanks, and 3 crude oil distillation units over 15 time intervals. The model contained 105 binary variables, 991 continuous variables, and 2154 constraints and was effectively solved with the proposed solution approach.

Warner and Prawda (1972) defined The Nursing Personnel Scheduling Problem as the identification of that staffing pattern which (1) specifies the number of nursing personnel of each skill class to be scheduled among the wards and nursing shifts of a scheduling period, (2) satisfies total nursing personnel capacity, integral assignment, and other relevant constraints, and (3) minimizes a "shortage cost" of nursing care services provided for the scheduling period. The problem was posed as a mixed-integer quadratic programming problem. The model was tested on six wards of a 600-bed general hospital, and results were presented.

Lai et al. (1992) developed an auxiliary multiple objective linear programming model to solve a linear programming problem with inaccurate objective and/or constraint coefficients. The objective function was to maximize the most possible value of the indefinite profit. At the same time, they tried to minimize the risk of obtaining lower profit and maximize the possibility of obtaining higher profit. This strategy is equivalent to the practical considerations of financial problems. A numeric investment problem was solved to illustrate this new approach.

Lawrie (1969) described an approach based on larger items of departments, group of pupils (generally year groups), and layouts. The problem was given an integer linear programming formulation, and computational methods used in obtaining solutions were discussed.

Ceselli et al. (2009) presented an optimization algorithm developed for a provider of software-planning tools for distribution logistics companies.

The algorithm computes a daily plan for a heterogeneous fleet of vehicles that depart from different depots and must visit a set of customers for delivery operations. The authors developed a column generation algorithm and described how to encode the cost function and the complicating constraints by an appropriate use of resources. They presented computational results on real instances obtained from the software company.

Aneja et al. (1979) developed a method of finding the extreme points that can be controlled in a criteria space. Such extreme points in a criteria space would be generally less and only these are needed while choosing a non-dominated solution for implementation. The method involved a parametric search in the criteria space. Although the method is developed with respect to a bicriteria transportation problem, it is applicable to any bicriteria linear program in general. A numerical example was included.

Klingman and Russell (1975) presented a specialized method for solving transportation problems with several additional linear constraints. The method is basically the simplex method, specialized to develop fully the structure embedded in the problem. The solution procedure required the storage of a spanning tree and a matrix for each basis.

Maio and Roveda (1971) considered a special class of transportation problems. These problems had a set of sources producing the same material with a fixed maximum capacity, and a set of users whose

demands for the material are known. A cost is associated with the transportation of the material from each source to each user. Each user is to be supplied by one source only. The problem was concerned with finding the flow of the material from the sources to the users that satisfies their demands and minimizes the total transportation cost. The paper proposed a search method for the solution of the problem, and discussed it from a computational point of view. The model was applied to a real industrial problem.

Al-Yakoob (1997) explored mathematical programming optimization models and algorithms for routing and scheduling ships in a maritime transportation system. The case study for the research was the Kuwait Petroleum Corporation (KPC) Problem. A mixed-integer programming model for the KPC problem was developed. The resulting mathematical formulation was complex to solve. Due to this complexity the model was revised. The problem was solved using CPLEX 4.0 MIP. The results gained were substantially better than the results obtained before the introduction of the model.

Klingman et al. (1974) described the development, implementation, and availability of a computer program for generating a variety of feasible network problems together with a set of benchmarked problems derived from it. In this work, a code "NETGEN" can generate capacitated and incapacitated transportation and minimum cost flow network problems, and assignment problems. In addition to generating structurally different classes of network problems the code permits the user to vary structural

characteristics within a class. This code is used to obtain the solution time and objective function value of 40 assignment, transportation, and network problems varying in size from 200 nodes to 8,000 nodes and from 1,300 arcs to 35,000 arcs.

In the next chapter, a linear programming model is formulated for the TOR transportation problem. To formulate the model, an objective function, decision variables and constraints for the transportation problem will be defined.

Chapter III: MODEL DEVELOPMENT

3.1 Introduction

In this chapter, a linear programming model will be developed. Many transportation and logistics problems faced by businesses fall into a category of problems known as network flow problems. A common scenario of a network flow problem arising in industrial logistics concerns the distribution of a single homogeneous product from plants (origins) to consumer markets (destinations). The total number of units produced at each plant and the total number of units required at each market are assumed to be known. The product need not be sent directly from source to destination, but may be routed through intermediary points reflecting warehouses or distribution centres. Further, there may be capacity restrictions that limit some of the shipping links. The objective of network flow problems is mainly to minimize the variable cost of producing and shipping the products to meet the consumer demand.

The distribution of oil-related products by TOR falls within the scenario given above. For the purpose of this paper, the following scenario will be considered;

> TOR has three depots scattered around the country in the cities of Kumasi, Accra and Takoradi. TOR has 40 tankers of gasoline at the depot in Takoradi, 50 tankers at the depot in Accra and 45 tankes at the depot in Kumasi. TOR supplies gasoline to five OMC's; Shell, GOIL, Total, Galaxy and ENGEN with demands of 40, 35, 30, 25 and 25

tankers respectively. Cost is incurred when fuel is transported from a depot to an OMC. Table 3.1 below summarises estimated distribution costs (in GH¢) between depots and OMCs.

Depot	Shell	Total	GOIL	Galaxy	ENGEN
Takoradi	30	40	32	29	33
Accra	26	20	24	30	22
Kumasi	21	25	34	50	42

Table 3.1: Estimated Distribution Costs between Depots and OMCs

3.2 Problem Formulation

In this section, a linear programming formulation of the TOR problem is developed. The decision variables, constraints and objective function of the problem are also presented.

To formulate the problem, the following terms are defined:

a_i = Number of units available at source i (i = 1, 2, . . . ,m);

b_j = Number of units required at destination j (j = 1, 2, . . . , n);

c_{ij} = Unit transportation cost from source i to destination j

$(i = 1, 2, . . . , m; j = 1, 2, . . . , n)$.

x_{ij} = Number of units to be distributed from source i to destination j

$(i = 1, 2, . . . , m; j = 1, 2, . . . , n)$,

The transportation problem is formulated as follows:

Minimise Z =

Subject to:

Expression (1) represents the minimization of the total distribution cost, assuming a linear cost structure for transporting.

Equation (2) states that the amount being transported from source i to all possible destinations should be less than or equal to the total availability, a_i, at that source.

Equation (3) indicates that the amounts being transported to destination j from all possible sources should be equal to the requirements, b_j, at that destination.

Equation (4) indicates the non-negativity constraint which ensures only positive amounts are transported from source i to destination j.

3.2.1 The Decision variables for TOR Problem

TOR needs to determine the least cost of transporting petrol from the various depots to the OMC's. A graphical representation of the problem is shown in Figure 3.1 below. The circles (or nodes) in the figure represent the depots and OMC's in the problem. The arrows (or arcs) connecting the various depots and OMC's represent different transportation routes. The decision problem faced by TOR is to determine how many tankers to transport on each of these routes. Hence, each of the arcs in the LP model represents a decision variable.
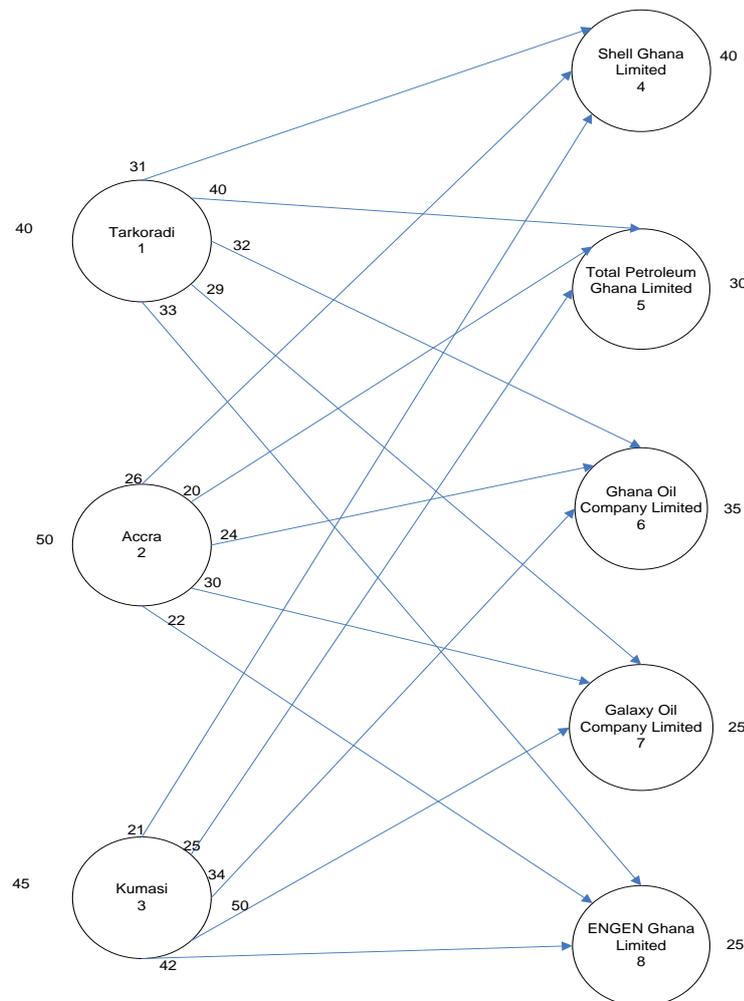
Figure 3.1: Network Representation of the TOR Transportation Problem

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In Figure 3.1, the number 1 identifies the node for Takoradi, 2 identifies the node for Accra and so on. For each arc in the LP model, I defined one decision variable as:

X_{ij} = the number of tankers transported from node i to node j

The network for TOR in Figure 3.1 has 15 arcs; hence, the LP formulation for this model requires the following 15 decision variables:

X_{14} = the number of tankers transported **from** node 1(Takoradi) **to** node 4(Shell)

X_{15} = the number of tankers transported **from** node 1(Takoradi) **to** node 5(Total)

X_{16} = the number of tankers transported **from** node 1(Takoradi) **to** node 6(GOIL)

X_{17} = the number of tankers transported **from** node 1(Takoradi) **to** node 7(Galaxy)

X_{18} = the number of tankers transported **from** node 1(Takoradi) **to** node 8(ENGEN)

X_{24} = the number of tankers transported **from** node 2 (Accra) **to** node 4(Shell)

X_{25} = the number of tankers transported **from** node 2(Accra) **to** node 5(Total)

X_{26} = the number of tankers transported **from** node 2(Accra) **to** node 6(GOIL)

X_{27} = the number of tankers transported *from* node 2(Accra) *to* node 7(Galaxy)

X_{28} = the number of tankers transported *from* node 2(Accra) *to* node 8(ENGEN)

X_{34} = the number of tankers transported *from* node 3(Kumasi) *to* node 4(Shell)

X_{35} = the number of tankers transported *from* node 3(Kumasi) *to* node 5(Total)

X_{36} = the number of tankers transported *from* node 3(Kumasi) *to* node 6(GOIL)

X_{37} = the number of tankers transported *from* node 3(Kumasi) *to* node 7(Galaxy)

X_{38} = the number of tankers transported *from* node 3(Kumasi) *to* node 8(ENGEN)

3.2.2 The Objective function for TOR Problem

Each tanker that travels from node *i* to node *j* in the network flow model in Figure 3.1 incurs some cost, C_{ij}. This cost represents the total cost, hence, making this problem a minimum cost network flow problem. Because this paper is geared toward minimising the total distribution costs, the objective function this problem is expressed as:

$$\text{MINIMISE} : 30X_{14} + 40X_{15} + 32X_{16} + 29X_{17} + 33X_{18} +$$

$$26X_{24} + 20X_{25} + 24X_{26} + 30X_{27} + 22X_{28} +$$

$$21X_{34} + 25X_{35} + 34X_{36} + 50X_{37} + 42X_{38}$$

The term $31X_{14}$ in this function reflects the fact that each tanker transported from Takoradi (node 1) to Shell (node 4) must incur a distribution cost of GH 31. The other terms in the function state similar relationships for the other transportation routes.

3.2.3 The Constraints for TOR Problem

The number of nodes in the network model determines the number of constraints in the LP formulation of the TOR network flow problem. Two physical constraints apply to the TOR problem. First, there is a limit on the number of tankers that can be transported to each OMC. TOR can transport no more than 40, 30, 35, 25, and 25 tankers to Shell, Total, GOIL, Galaxy and ENGEN respectively. These restrictions are represented by the constraints below:

$$X_{14} + X_{24} + X_{34} \leq 40 \qquad \text{\} demand for Shell}$$

$$X_{15} + X_{25} + X_{35} \leq 30 \qquad \text{\} demand for Total}$$

$$X_{16} + X_{26} + X_{36} \leq 35 \qquad \text{\} demand for GOIL}$$

$$X_{17} + X_{27} + X_{37} \leq 25 \qquad \text{\} demand for Galaxy}$$

$$X_{18} + X_{28} + X_{38} \leq 25 \qquad \text{\} demand for ENGEN}$$

The first constraint shows that the total number of tankers transported from Takoradi (node 1), Accra (node 2) and Kumasi (node 3) to Shell (node 4) must be less than or equal to Shell's demand of 40 tankers. This is because; the total demand (155 tankers) is more than the total supply (135 tankers) at the depots. The other constraints give a similar interpretation for the other OMC's.

The second set of constraints ensures that the supply of gasoline at each depot is transported to an OMC. This is represented by the constraints below:

$X_{14} + X_{15} + X_{16} + X_{17} + X_{18} = 40$ } supply available at Takoradi

$X_{24} + X_{25} + X_{26} + X_{27} + X_{28} = 50$ } supply available at Accra

$X_{34} + X_{35} + X_{36} + X_{37} + X_{38} = 45$ } supply available at Kumasi

The first constraint shows that the total number of tankers transported from Takoradi (node 1) to Shell (node 4), Total (node 5), GOIL (node 6), Galaxy (node 7) and ENGEN (node 8) must be equal to the total amount available. The other constraints give a similar interpretation for the other depots.

3.2.4 Implementing the Model in a Spreadsheet

The LP model for the TOR problem is summarised as:

$$\text{MIN:} \quad 30X_{14} + 40X_{15} + 32X_{16} + 29X_{17} + 33X_{18} +$$
$$26X_{24} + 20X_{25} + 24X_{26} + 30X_{27} + 22X_{28} +$$
$$21X_{34} + 25X_{35} + 34X_{36} + 50X_{37} + 42X_{38}$$

Subject to:

$$X_{14} + X_{24} + X_{34} \leq 40 \qquad \} \text{ demand for Shell}$$

$$X_{15} + X_{25} + X_{35} \leq 30 \qquad \} \text{ demand for Total}$$

$$X_{16} + X_{26} + X_{36} \leq 35 \qquad \} \text{ demand for GOIL}$$

$$X_{17} + X_{27} + X_{37} \leq 25 \qquad \} \text{ demand for Galaxy}$$

$$X_{18} + X_{28} + X_{38} \leq 25 \qquad \} \text{ demand for ENGEN}$$

$$X_{14} + X_{15} + X_{16} + X_{17} + X_{18} = 40 \quad \} \quad \text{supply available at}$$
Takoradi

$$X_{24} + X_{25} + X_{26} + X_{27} + X_{28} = 50 \quad \} \text{ supply available at Accra}$$

$$X_{34} + X_{35} + X_{36} + X_{37} + X_{38} = 45 \quad \} \text{ supply available at Kumasi}$$

$$X_{ij} \geq 0, \text{ for all } i \text{ and } j \qquad \} \text{ non-negativity constraint}$$

The last constraint indicates that all the decision variables must be nonnegative.
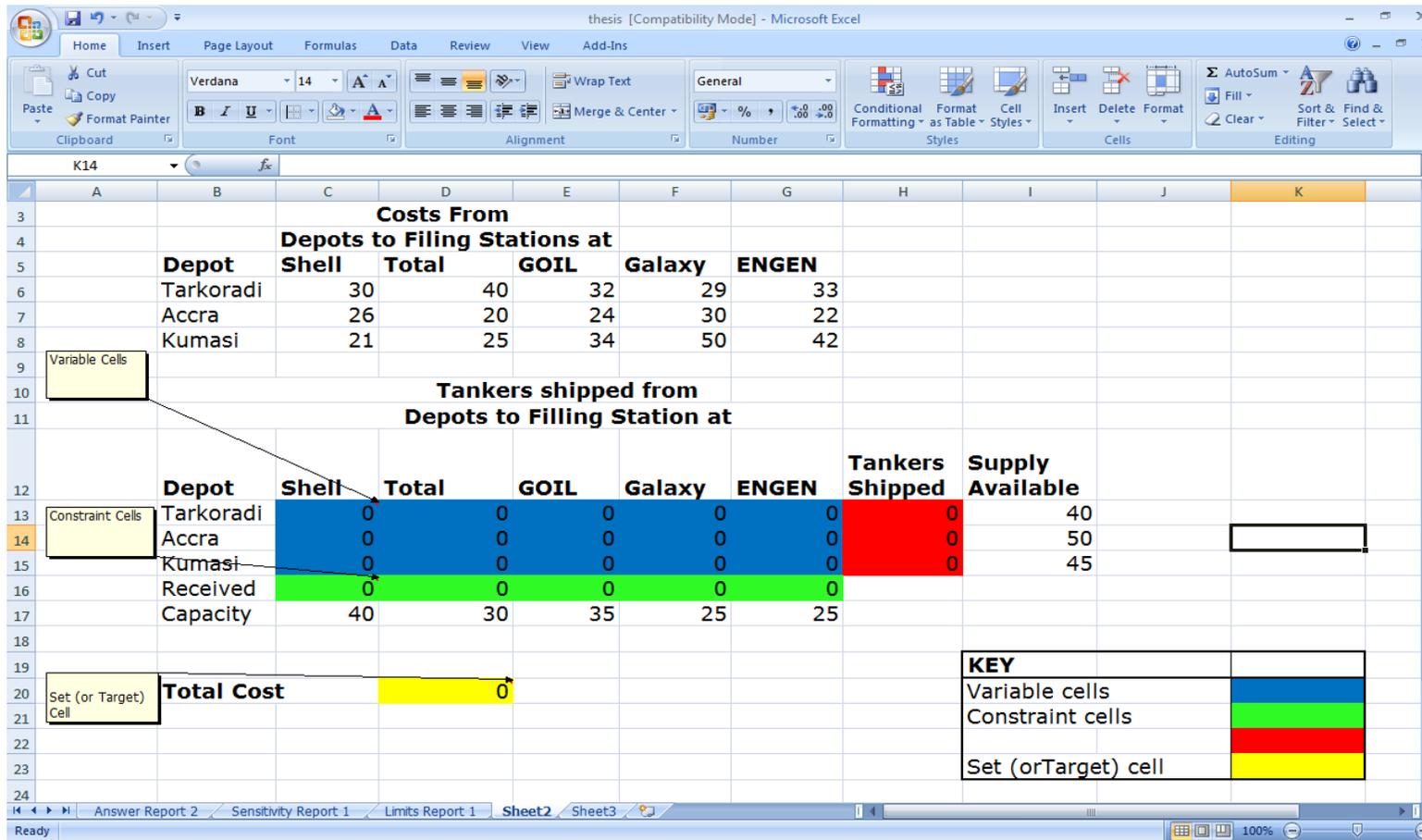
Figure 3.2: Spreadsheet model for TOR transportation problem

3.2.4.1 Key Cell Formulas

In the spreadsheet, the costs between each depot and OMC are represented in Cells C6 to G8. Cells C13 to G15 are reserved for representing the number of tankers of gasoline to transport form each depot to each OMC.

The left hand side formulas for the five capacity constraints in the model are implemented in cells C16, D16, E16, F16 and G16 in the spreadsheet. The formula entered in cell C16 and then copied to cells D16, E16, F16 and G16 is as follows:

Formula for cell C16   =SUM (C13:C15)

These cells represent the total tankers of gasoline being transported to Shell, Total, GOIL, Galaxy and ENGEN respectively. Cells C17 to G17 contain the right hand side values for these constraint cells.

The left hand formulas for the three supply constraints in the model are implemented in the cells H13, H14 and H15 as:

Formula for cell H13   =SUM (C13:G13)

These cells represent the total tankers of gasoline being shipped from the BOST depots at Takoradi, Accra and Kumasi respectively. Cells I13 to I16 contain the right hand side values for these constraint cells.

The objective function for this model is entered in the cell D20 as:

Formula for cell D20   =SUMPRODUCT (C6:G9,C13:G15)

The SUMPRODUCT function multiplies each element in the range C6 to G8 by the corresponding element in the range C13 to G15 and then sums the individual products.

Cell	Formula	Copied To
D20	=SUMPRODUCT(C6:G9,C13:G15)	-
C16	=SUM(C13:C15)	D16:G16
H13	=SUM(C13:G13)	H14:H15

Table 3.2: Summary of Key Cell Formulas

In the next chapter, we present the computational results for TOR problem as well as solution strategies.

Chapter IV: COMPUTATIONAL RESULTS

4.1 Introduction

Solution strategies and relevant computational results for the problem formulation of TOR are presented in this chapter. This chapter is organized as follows. In Section 4.2, we solve the problem using Solver. Solver is an in-built optimisation tool in Microsoft Excel. Microsoft Excel spreadsheets are the most common and one of the most effective ways to manage business and supply chain operations. They provide the flexibility to address the issues unique to your business in a standard and easy to understand format. The results provided by Solver is analysed in Section 4.3.

4.2 Solving the Model

After implementing the model in the spreadsheet, we use Solver (an in-built spreadsheet optimisation tool) to find the optimal solution to the problem. To do this, we need to specify the set (or target) cell, variable cells and constraints cells identified in Figure 3.2 in Chapter 3. The set (or target) cell is the cell in the spreadsheet that represents the objective function in the model. Variable cells are the cells in the spreadsheet that represent the decision variables in the model. Constraint cells are the cells in the spreadsheet that represent the left hand formulas of the constraints in the model (and any upper and lower bounds that apply to these formulas).

4.2.1 Defining the Set (or Target) Cell

We specify in Solver, the location of the set cell as D20 as shown in Figure 4.1. Cell D20 contains the formula representing the objective function for the TOR problem and we select the Min button because the objective is to minimise total distribution costs.
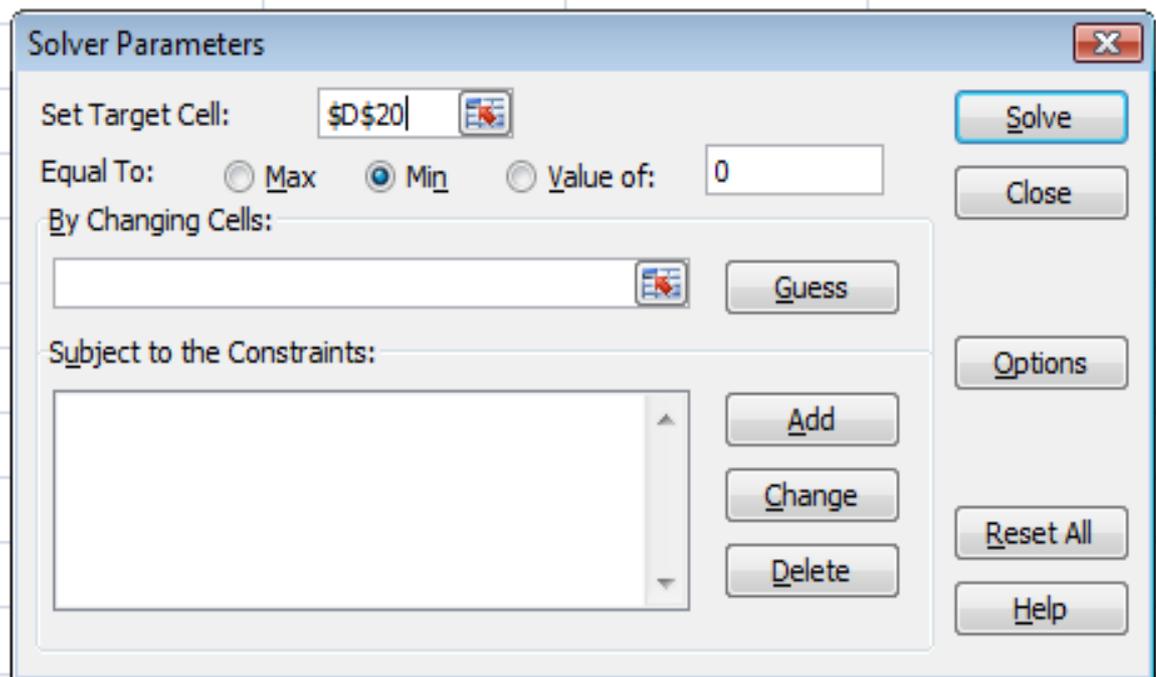
Figure 4.1: Specifying the set (or target) cell

4.2.2 Defining the Variable Cells

We need to specify which cells represent the decision variables in the model. Solver refers to these cells as variable cells. Cells C13 to G15 represent the decision variables for the model as shown in Figure 4.2 below. The optimal values for these cells will be determined by Solver.
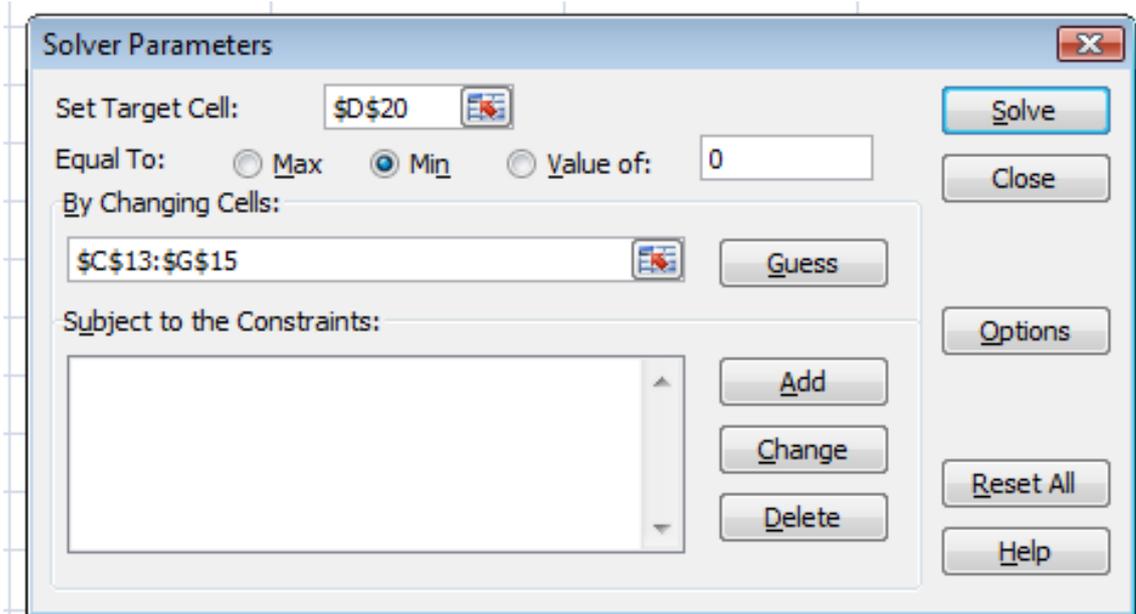
Figure 4.2: Specifying the variable (or changing) cells

4.2.3 Defining the Constraint Cells

The constraint cells are the cells in which we implemented the left hand side formulas for each constraint in the model, as mentioned earlier. Cells C16 to G16 represent constraint cells whose values must be less than or equal to the values in cells C17 to G17 respectively. Cells H13 to H15 represent constraint cells whose values must be equal to the values in cells H13 to H15 respectively. These constraints are shown in figure 4.3 below.
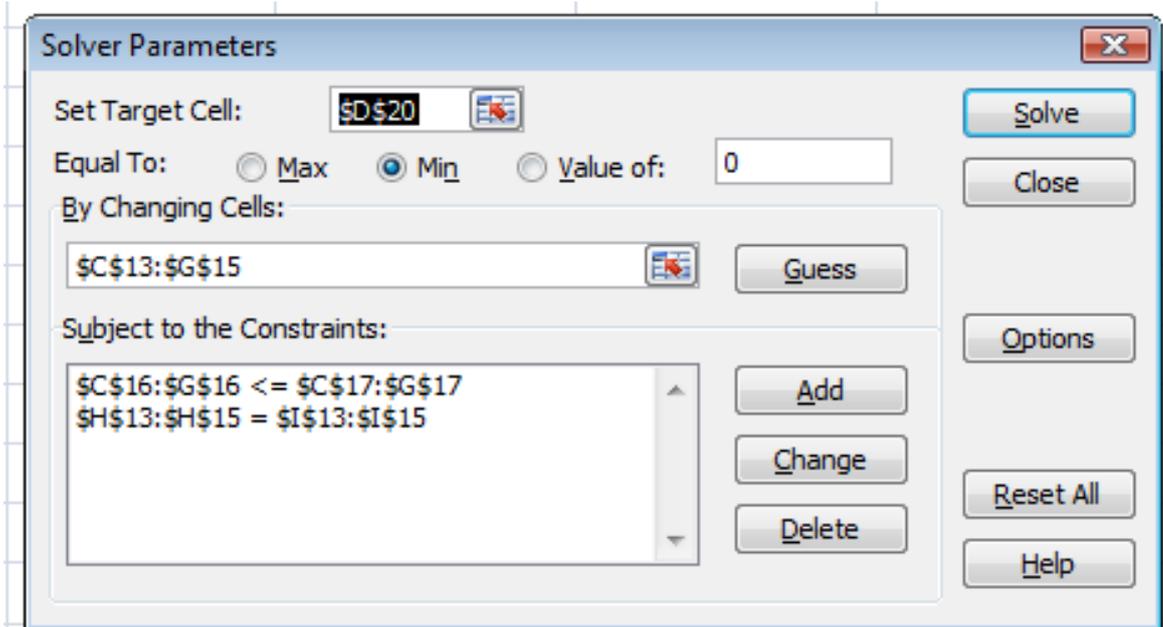
Figure 4.3: Specifying the constraint cells

4.2.4 Defining the Non-negativity Conditions

The non-negativity conditions ensure that the values of the decision variables are not negative. In the model, it has been indicated that the decision variables must be greater than or equal to zero. To specify this in Solver, I indicate that cells C13 through G15 must be greater than or equal to zero. This is shown in Figure 4.4 below.
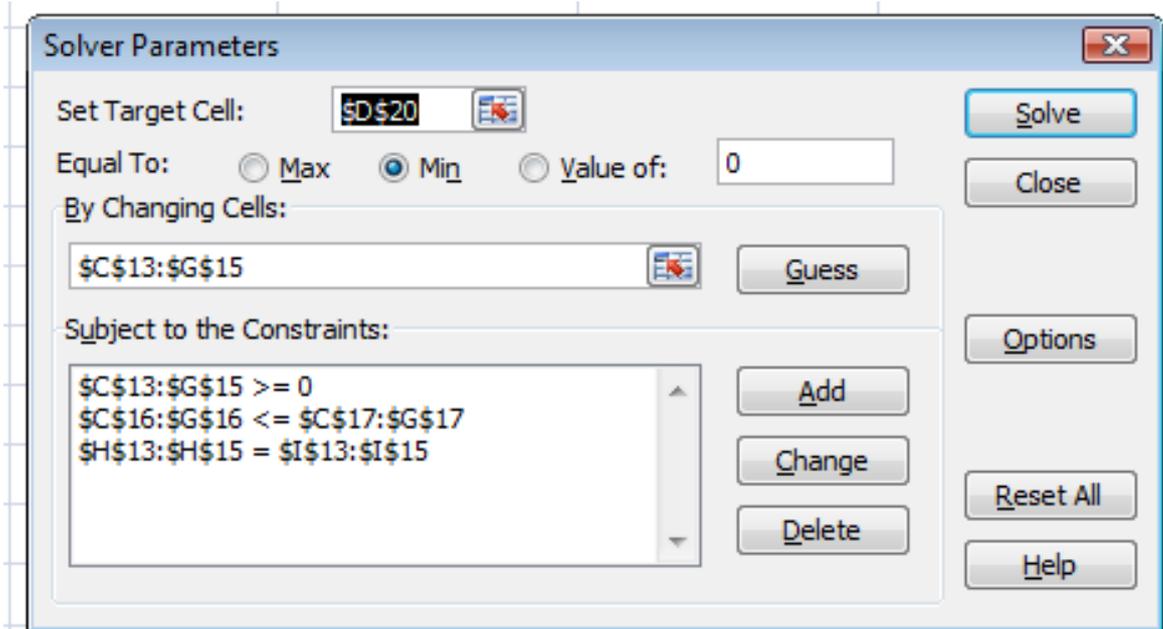
Figure 4.4: Specify the non-negativity constraints

4.2.5 Specifying Linear Model Option

After specifying the set/target cell, variable cells, constraint cells and the non-negativity constraint, the "Assume Linear Model" option is selected as shown in Figure 4.5 below. This option indicates to Solver that the problem to be solved is of a linear nature.
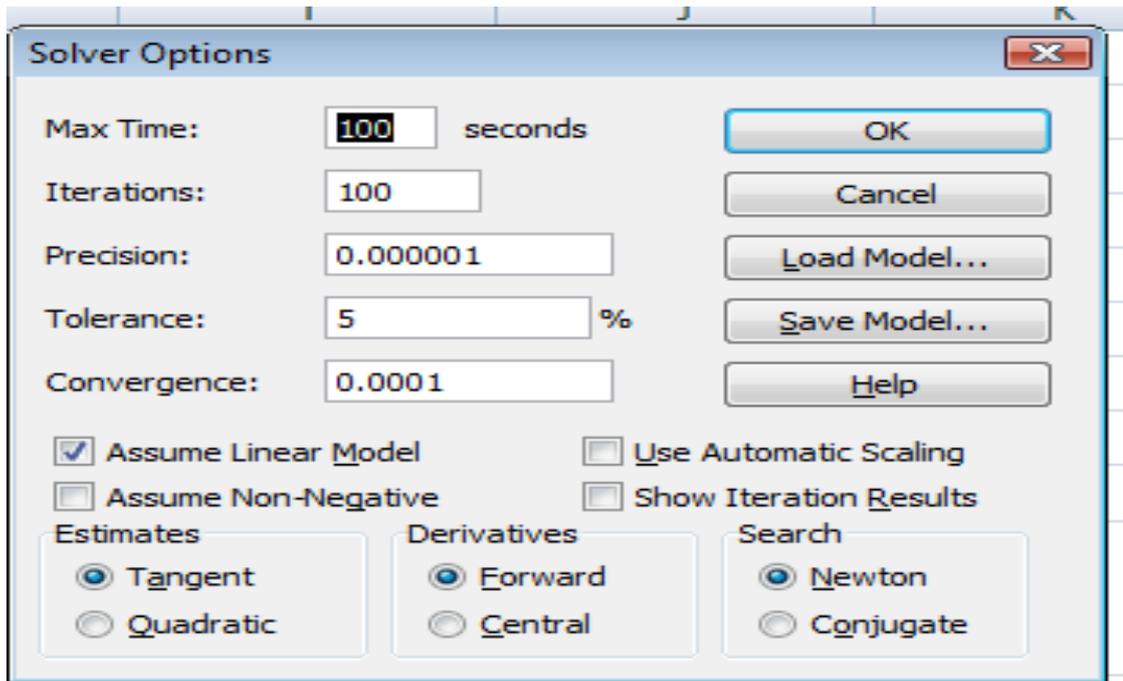
Figure 4.5: Specify Linear Model Option

4.2.6 Solving the Model

After specifying all the appropriate parameters, the problem is then solved by clicking on the solve button. As shown in Figure 4.6, Solver determined that the optimal values for cells C13, C14, C15, D13, D14, D15, E13, E14, E15, F13, F14, F15, G13, G14, and G15 are 0, 0, 40, 0, 25, 5, 15, 0, 0, 25, 0, 0, 0, 25 and 0 respectively. The value of the set (or target) cell now indicates 3220 which is the total distribution cost.
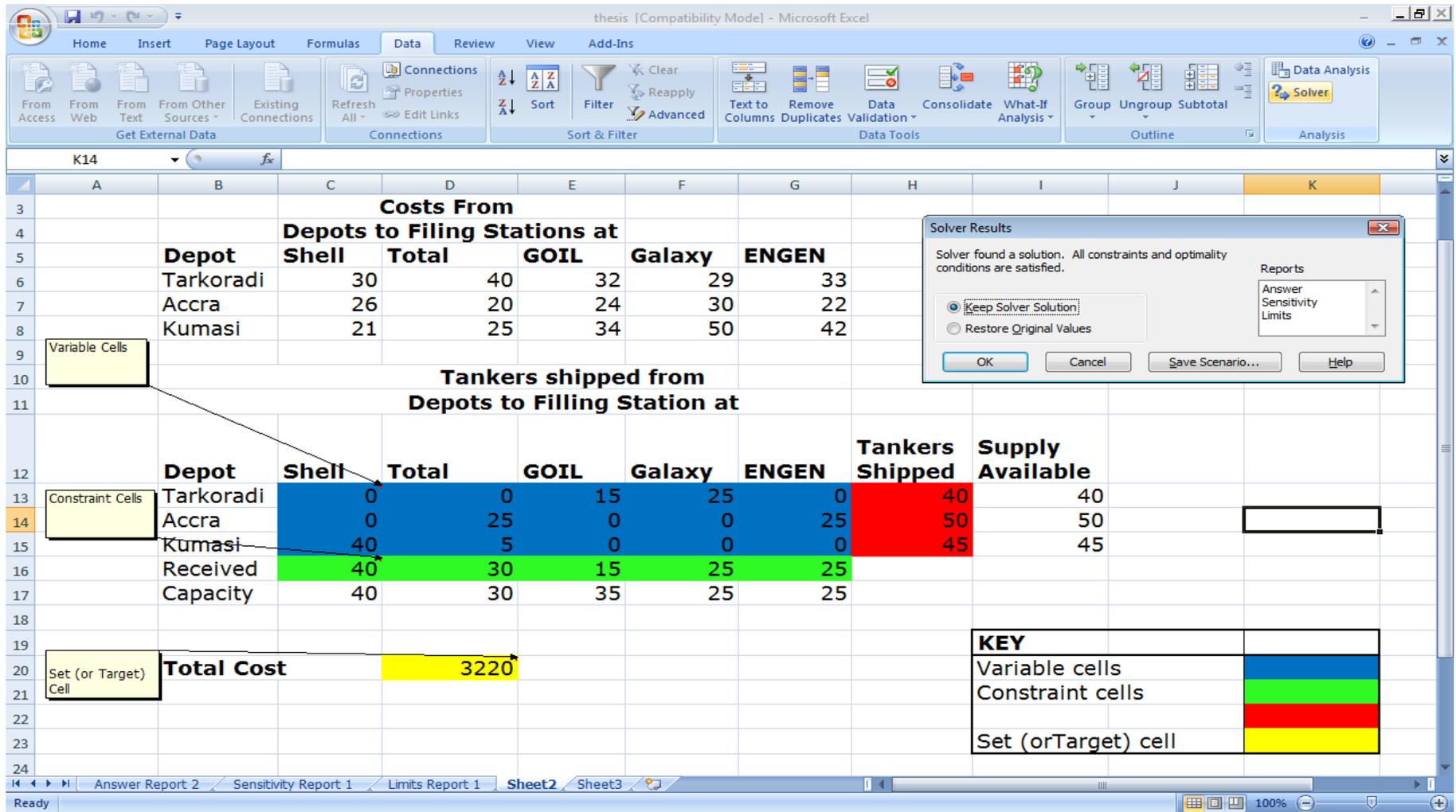
Figure 4.5: Optimal solution to TOR transportation problem

4.3 Analyzing the solution

The optimal solution in Figure 4.6 indicates that 15 tankers should transport from Takoradi to GOIL (X_{16} = 15) and 25 tankers should transport from Takoradi to Galaxy (X_{17} = 25). Of the 50 tankers available at the depot in Accra, 25 tankers must transport to Total (X_{25} = 25) and 25 tankers to ENGEN (X_{28} = 25). Finally, of the 45 tankers available at the depot in Kumasi, 40 tankers must transport to Shell (X_{34} = 40) and 5 tankers to Total (X_{35} = 5). The solution showed in Figure 4.5 satisfies all the constraints in the model and results in a minimum distribution cost of GH¢ 3,220.00

4.3.1 Sensitivity Analysis

Sensitivity analysis will provide a better picture of how the solution given by Solver will change if different factors in the model change. This is because the solution given by Solver is not necessarily correct given the fact that certain variables when changed will affect or impact on the optimal solution. Solver provided some sensitivity information after solving the LP problem. This information included the answer report, the sensitivity report and the limit reports. The information in these reports will be discussed subsequently.

4.3.1.1 The Answer Report

The answer report summarises the solution to the TOR problem and it's fairly self-explanatory. This report comprises of three sections. Figure 4.7.1 shows the first two sections of the report and Figure 4.7.2 shows the third section of the report. The first section of the report summarises the original and optimal values of the set (or target) cell representing the objective function. The second section of the report summarises the original and optimal values of the adjustable (or changing) cells representing the decision variables. The final section of this report gives information about the constraints.

The Cell Value column shows the optimal value assumed by each constraint cell. The Formula column shows the upper or lower bounds that apply to each constraint cell. The Status column shows which constraints are binding and those which are nonbinding. A constraint is binding if it is satisfied as a strict equality in the optimal solution, else, it is nonbinding. Thus, from this report we can see that Shell, Total, Galaxy and ENGEN get all their demands met. The Slack column shows the difference between the left hand side and the right hand side of each constraint. The values in the Slack column show that if this solution is implemented, all the OMCs will have all their demands met except GOIL. Finally, the slack values for the non-negativity conditions indicate the amounts by which the decision variables exceed their lower bounds of zero.
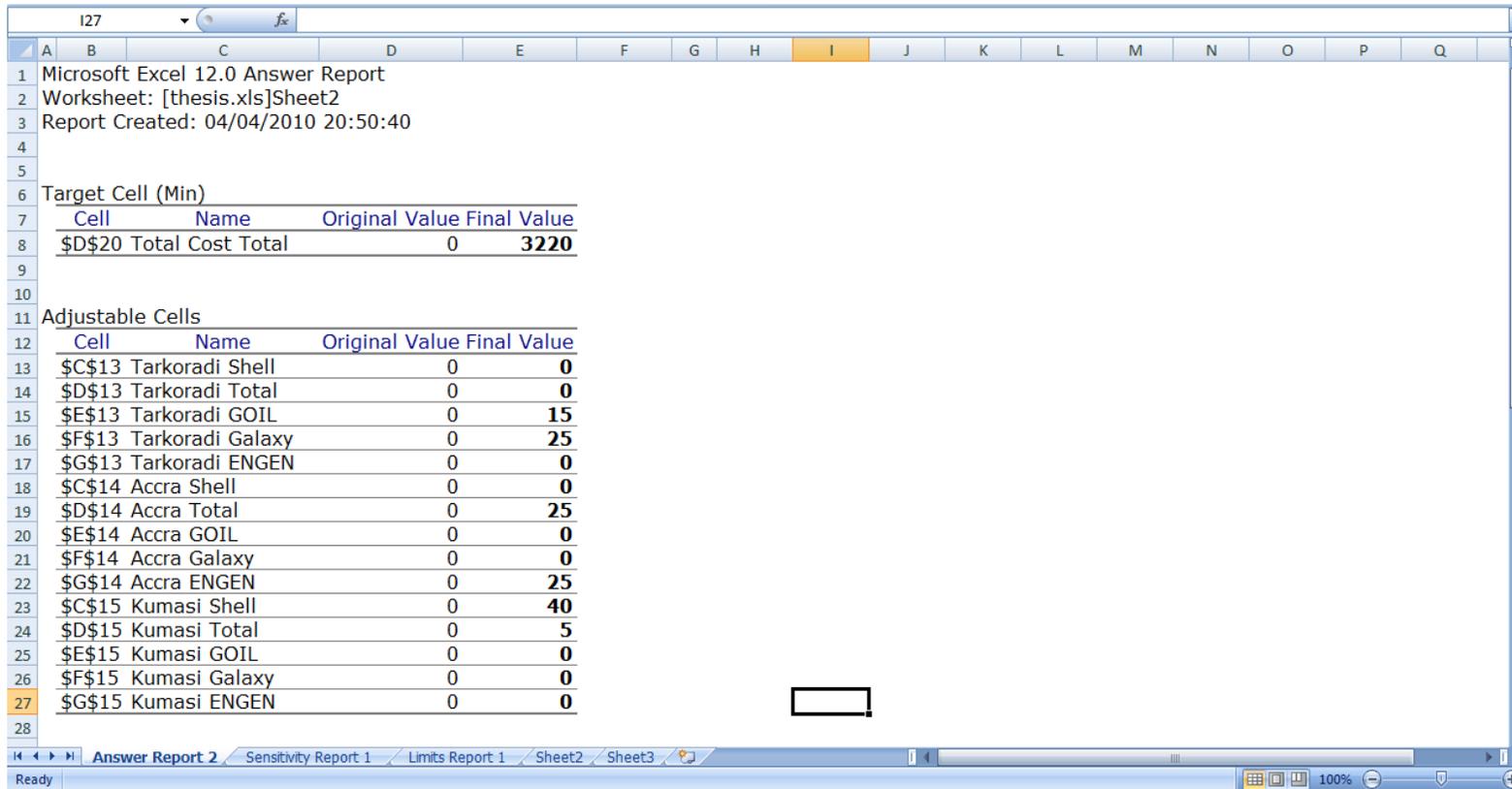
Figure 4.6.1: Answer report for TOR problem

O57

	A	B	C	D	E	F	G
29							
30	**Constraints**						
31		Cell	Name	Cell Value	Formula	Status	Slack
32		$C$16	Received Shell	40	$C$16<=$C$17	Binding	0
33		$D$16	Received Total	30	$D$16<=$D$17	Binding	0
34		$E$16	Received GOIL	15	$E$16<=$E$17	Not Binding	20
35		$F$16	Received Galaxy	25	$F$16<=$F$17	Binding	0
36		$G$16	Received ENGEN	25	$G$16<=$G$17	Binding	0
37		$H$13	Tarkoradi Tankers Shipped	40	$H$13=$I$13	Not Binding	0
38		$H$14	Accra Tankers Shipped	50	$H$14=$I$14	Not Binding	0
39		$H$15	Kumasi Tankers Shipped	45	$H$15=$I$15	Not Binding	0
40		$C$13	Tarkoradi Shell	0	$C$13>=0	Binding	0
41		$D$13	Tarkoradi Total	0	$D$13>=0	Binding	0
42		$E$13	Tarkoradi GOIL	15	$E$13>=0	Not Binding	15
43		$F$13	Tarkoradi Galaxy	25	$F$13>=0	Not Binding	25
44		$G$13	Tarkoradi ENGEN	0	$G$13>=0	Binding	0
45		$C$14	Accra Shell	0	$C$14>=0	Binding	0
46		$D$14	Accra Total	25	$D$14>=0	Not Binding	25
47		$E$14	Accra GOIL	0	$E$14>=0	Binding	0
48		$F$14	Accra Galaxy	0	$F$14>=0	Binding	0
49		$G$14	Accra ENGEN	25	$G$14>=0	Not Binding	25
50		$C$15	Kumasi Shell	40	$C$15>=0	Not Binding	40
51		$D$15	Kumasi Total	5	$D$15>=0	Not Binding	5
52		$E$15	Kumasi GOIL	0	$E$15>=0	Binding	0
53		$F$15	Kumasi Galaxy	0	$F$15>=0	Binding	0
54		$G$15	Kumasi ENGEN	0	$G$15>=0	Binding	0
55							
56							

Answer Report 2 / Sensitivity Report 1 / Limits Report 1 / Sheet2 / Sheet3

Ready   100%

Figure 4.7.2: Answer report for TOR problem

4.3.1.2 The Sensitivity Report

The sensitivity report summarises information about the variable cells and constraints for the model. The values in the Objective Coefficient column in Figure 4.8.1 show the original objective function coefficients, that is, the distribution cost of each decision variable. The Allowable Increase and Allowable Decrease column show the allowable increases and decreases in the distribution costs. For instance, the distribution cost from Takoradi to GOIL can increase as much as GH¢ 1 or decrease as much as GH¢ 3 without changing the optimal solution. This is under the assumption that all the other distribution costs remain constant.

The shadow price for a constraint identifies the amount by which the objective function value changes given a unit increase in the right hand side value of the constraints, under the assumption that all other distribution costs remain constant. For instance, Figure 4.8.2 below shows that Shell has a shadow price of -6. This means that if the number of tankers transported to Shell is reduced from 40 to 39, the total distribution cost will reduce by GH¢ 6.

	R11			fx														
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S

	A	B	C	D	E	F	G	H
1	Microsoft Excel 12.0 Sensitivity Report							
2	Worksheet: [thesis.xls]Sheet2							
3	Report Created: 04/04/2010 20:51:55							
4								
5								
6	Adjustable Cells							
7				Final	Reduced	Objective	Allowable	Allowable
8		Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9		$C$13	Tarkoradi Shell	0	4	30	1E+30	4
10		$D$13	Tarkoradi Total	0	10	40	1E+30	10
11		$E$13	Tarkoradi GOIL	15	0	32	1	3
12		$F$13	Tarkoradi Galaxy	25	0	29	3	1E+30
13		$G$13	Tarkoradi ENGEN	0	1	33	1E+30	1
14		$C$14	Accra Shell	0	10	26	1E+30	10
15		$D$14	Accra Total	25	0	20	2	7
16		$E$14	Accra GOIL	0	2	24	1E+30	2
17		$F$14	Accra Galaxy	0	11	30	1E+30	11
18		$G$14	Accra ENGEN	25	0	22	2	2
19		$C$15	Kumasi Shell	40	0	21	4	1E+30
20		$D$15	Kumasi Total	5	0	25	7	4
21		$E$15	Kumasi GOIL	0	7	34	1E+30	7
22		$F$15	Kumasi Galaxy	0	26	50	1E+30	26
23		$G$15	Kumasi ENGEN	0	15	42	1E+30	15
24								
25	Constraints							
26				Final	Shadow	Constraint	Allowable	Allowable
27		Cell	Name	Value	Price	R.H. Side	Increase	Decrease
28		$C$16	Received Shell	40	-6	40	5	0

Answer Report 2	Sensitivity Report 1	Limits Report 1	Sheet2	Sheet3

Ready   100%

Figure 4.8.1: Sensitivity report for TOR problem

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Cell	Name	Value	Cost	Coefficient	Increase	Decrease
$C$13	Tarkoradi Shell	0	4	30	1E+30	4
$D$13	Tarkoradi Total	0	10	40	1E+30	10
$E$13	Tarkoradi GOIL	15	0	32	1	3
$F$13	Tarkoradi Galaxy	25	0	29	3	1E+30
$G$13	Tarkoradi ENGEN	0	1	33	1E+30	1
$C$14	Accra Shell	0	10	26	1E+30	10
$D$14	Accra Total	25	0	20	2	7
$E$14	Accra GOIL	0	2	24	1E+30	2
$F$14	Accra Galaxy	0	11	30	1E+30	11
$G$14	Accra ENGEN	25	0	22	2	2
$C$15	Kumasi Shell	40	0	21	4	1E+30
$D$15	Kumasi Total	5	0	25	7	4
$E$15	Kumasi GOIL	0	7	34	1E+30	7
$F$15	Kumasi Galaxy	0	26	50	1E+30	26
$G$15	Kumasi ENGEN	0	15	42	1E+30	15

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$16	Received Shell	40	-6	40	5	0
$D$16	Received Total	30	-2	30	25	0
$E$16	Received GOIL	15	0	35	1E+30	20
$F$16	Received Galaxy	25	-3	25	15	20
$G$16	Received ENGEN	25	0	25	1E+30	0
$H$13	Tarkoradi Tankers Shipped	40	32	40	20	15
$H$14	Accra Tankers Shipped	50	22	50	0	25
$H$15	Kumasi Tankers Shipped	45	27	45	0	5

Figure 4.7.2: Sensitivity report for TOR problem

4.3.1.3 The Limit Report

The limit report lists the optimal value of the set (or target) cell. It also summarises the optimal values for each variable cell and shows what values the set (or target) cell assumes if each variable cell is set to its upper or lower limits. Figure 4.9 below shows the Limit Report for the TOR problem. The values in the Lower Limit column indicate the smallest value each variable cell can assume while the values of the other variable cells remain constant and all the constraints are satisfied. The values in the Upper Limit column show the largest value each variable cell can assume while the values of the other variable cells remain constant and all the constraints are satisfied.
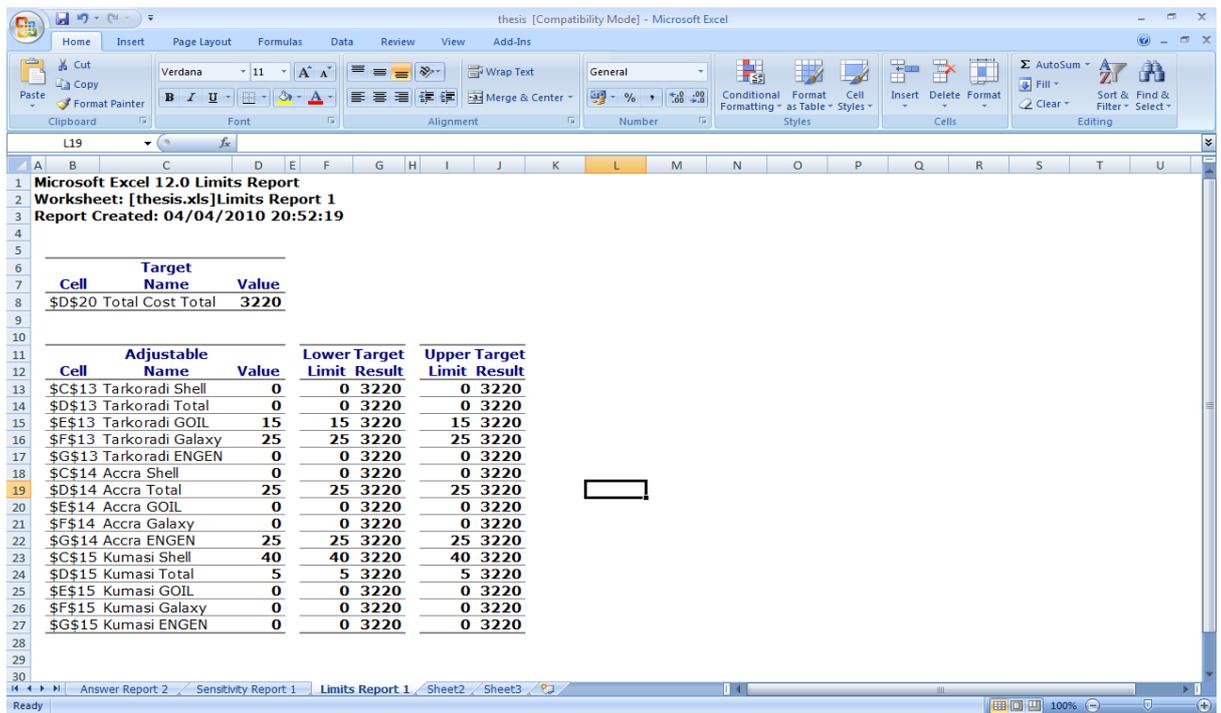

Figure 4.8: Limit report for TOR problem

Chapter V: CONCLUSION AND RECOMMENDATION

Planning and scheduling activities related to product distribution have been receiving growing attention for the past decade. Every company should focus on attending to all its clients' requirements at the lowest possible cost (Bodin et al., 1983) (See Section 1.1) and hence, emphasize the need for efficient and reliable land transportation systems. Efficient allocation of products has the potential of enormous savings in the total distribution costs.

The Tema Oil Refinery (TOR) transportation problem was the main focus for this research. A linear programming model for the TOR problem was formulated in this paper. This model takes into account an oil-related product; gasoline, three (3) BOST depots, five (5) OMC's, etc. In the process of formulating the transportation problem for TOR, we attempted to simulate the actual operation as closely as possible in order to produce a realistic model that can be utilized for product allocation in actual operation.

Finally, this proposed model can serve as a useful tool for gaining an edge in the negotiation process. By running the model in a sensitivity analysis fashion for various possible delivery options, TOR can pre-assess the effect of a given contract on its overall operations and net cost. Furthermore, the availability of sufficient computing power can facilitate the generation of new schedules regularly, easily, and at a very short notice, as need arises. This LP model can be modified to include all the other OMC's and depots. And also, the non-negativity constraint could set

to any integer greater than zero to ensure that all distribution routes, that

is, from one depot to an OMC, receive some gasoline.

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GLOSSARY

Algorithm: A step-by-step problem-solving procedure, especially an established, recursive computational procedure for solving a problem in a finite number of steps.

Allowable decrease: amount by which the coefficient of a decision variable can increase without changing the optimal solution assuming that all the other decision variable coefficients remain constant.

Allowable Increase: amount by which the coefficient of a decision variable can decrease without changing the optimal solution assuming that all the other decision variable coefficients remain constant.

Arcs: lines connecting nodes in a network flow problem

Auxiliary Multiple Objective Linear Programming Model: an LP model with multiple objective functions

Bicriteria Transportation Problem: a transportation problem with two or more relevant objective functions

Binary Variable: a variable that can assume only two integer values; 0 and 1

BOST: Bulk Oil Storage and Transport Company

Bpd: Barrel Per Day

Capacitated Transportation: a transportation problem with capacity constraints

Column Generation Algorithm: is an efficient algorithm for solving larger linear programming model by finding only the variables which have the potential to improve the objective function.

Constraint Cells: cells in the spreadsheet that represent the left hand formulas of the constraints in the model.

Constraint: is some function of the decision variables that must be less than or equal to, greater than or equal to or equal to some specific value.

Continuous Variables: variables that are not required to strictly assume integer values

Decision Variables: these represent the decisions that must be made

GOIL: Ghana Oil Company Limited

Heuristic Method: a technique for making decisions that might work well in some instances but it is not guaranteed to produce optimal solutions or decisions

Incapacitated Transportation: a transportation problem with no capacity constraints

Inductive Algorithm: an algorithm that is developed based on conclusions from observation

Combinatorial Algorithm: an algorithm that is related to probability and statistics

Linear inequality: a mathematical statement indicating that two quantities are not equal

Linear equality: a mathematical statement indicating that two quantities are equal

Linear Programming (LP): a mathematical programming technique which involves creating and solving optimization problems with linear objective functions and linear constraints (Ragsdale, 2004)

Mathematical Programming: a field of management science that finds the optimal, or most efficient way of using limited resources to achieve the objectives of an individual or a business (Ragsdale, 2004).

Maximal Flow Problem: a type of network flow problem in which the goal is to determine the maximum amount of flow that can occur in the network

Minimum Cost Network Flow Problem: a type of network flow problem in which the objective is to minimise cost, distance or penalty

Mixed-Integer Optimization Model: a model in which only some of the unknown variables are required to be integers

Mixed-Integer Quadratic Programming Problem: a problem in which the variables have exponential powers greater than one

Model: a representation of the reality that captures the essence of reality

Network Flow Problem: this is a problem of determining the amounts of a commodity to transport from a source to a destination subject to some constraints

Network Model: a mathematical representation of a network flow problem

Nodes: circles in a network flow problem

Non-negativity Condition: a condition that ensures that the values of the decision variables are not negative

Objective Function: it identifies some function of the decision variables that the decision maker wants to either minimise or maximise.

Oil Marketing Company (OMC): a company that acts as an intermediary between oil refineries and consumers

Optimal Solution: it is the best answer to the problem given the constraints

Optimization: to make something function at its best or most effective, or use something to its best advantage.

Parametric Search: A type of search that looks for objects that contain a numeric value or attribute, such as dates, integers, or other numeric data types

Probabilistic Constraint: a constraint that has a degree of certainty of occurrence

Set (Or Target) Cell: is the cell in the spreadsheet that represents the objective function in the model

Shadow price: it identifies the amount by which the objective function value changes given a unit increase in the right hand side value of the constraints, under the assumption that all other distribution costs remain constant.

Simplex Method: an efficient way of solving LP problems

Solver: an in-built optimisation tool in MSEXCEL

Stepping Stone Method: a method of solving transportation problems that adapt the simplex method by using the special structure of the problem.

Stochastic Programming Model: a programming model that involves uncertainty

TOR: Tema Oil Refinery

Transshipment Problem: is an extension of the transportation problem in which intermediate transshipment points are added between the sources and destinations.

Transportation/Assignment Problem: a type of network flow problem which aims to find the best way to fulfill the demand of n demand points using the capacities of m supply points at the minimum cost.

Variable cells: the cells in the spreadsheet that represent the decision variables in the model.