



ASHESI UNIVERSITY

DESIGN OF A RACK AND PINION ACKERMANN STEERING SYSTEM TO PREVENT CARJACKING

CAPSTONE PROJECT

B.Sc. Mechanical Engineering

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ASHESI UNIVERSITY

**DESIGN OF A RACK AND PINION ACKERMANN STEERING
SYSTEM TO PREVENT CARJACKING**

CAPSTONE PROJECT

Capstone Project submitted to the Department of Engineering, Ashesi

University in partial fulfilment of the requirements for the award of

Bachelor of Science degree in Mechanical Engineering.

Wilfred Amoo-Gottfried

2019

Declaration

I hereby declare that this applied project is the result of my own original work and that no part of it has been presented for another degree in this university or elsewhere.

Candidate's Signature:

.....

Candidate's Name:

.....

Date:

.....

I hereby declare that preparation and presentation of this applied project were supervised in accordance with the guidelines on supervision of applied project laid down by Ashesi University College.

Supervisor's Signature:

.....

Supervisor's Name:

.....

Date:

.....

Acknowledgement

I would like to extend my appreciation to every lecturer I have met during my time in Ashesi University as they have shaped me as a person during this period. I would also like to give a special thanks to my supervisor, Dr. Morris, who has guided me throughout this project.

Abstract

Rack and pinion gears are used in small sized vehicles due to their small size, lightness and ease of repair to produce steering systems. The main purpose of this paper is to design and implement a rack and pinion steering system using Ackermann steering conditions, on a go-kart, to aid in preventing carjacking. With the use of Solidworks, a virtual rack and pinion steering system is designed and simulated. Furthermore, the rack and pinion gears designed are 3D-printed and implemented onto an existing go-kart. Using Ackermann steering conditions, quantities such as the turning radius, Ackermann angle, outer and inner wheel angle, steering torque, etc. are calculated and practically tested on the go-kart. This is to help prevent the tyres on the go-kart from slipping when taking a turn. Additionally, a GPS tracking system and motors are installed onto the go-kart, with the motors being controlled via a mobile app. From simulation in Solidworks, it was shown that there were minimal contact stresses between the rack and pinion gears. It was observed that the structure would be void of failure as it had a factor of safety between 4.034 and 3.925×10^{10} . Lastly, from the practical verification of the Ackermann steering conditions calculated, all the angles measured with a protractor were similar. This verification demonstrated that the steering system designed adhered to the Ackermann steering conditions.

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1 CHAPTER 1: INTRODUCTION

1.1 Overview

Carjacking is a crime in which an individual takes another person's vehicle either forcefully or through acts of intimidation. In the act of carjacking, the victim is often left injured and in extreme cases, left dead [1]. Generally, several stories have been shared about carjacking and most of them sound unbelievable. Unfortunately, Ghana has been a hub of such cases.

According to a report from Graphic online, carjackers have adopted new tactics in carjacking. For instance, the carjackers can collaborate with mechanics to declare to a car owner that his car is faulty. The mechanic will then proceed to check the car and drive it on the pretense of understanding the fault. Eventually, the mechanic speeds away with the car. Such tactics increase the ease of theft for the carjackers [2]. Frankly, this story sounds absurd and it is quite shocking that such a scenario occurs in Ghana.

Also, in Winneba, it was found that two carjackers forcefully took a taxi driver's vehicle and ordered him to drive to the outskirts of the town. The driver had no option but to comply and was thrown out of his vehicle in the end. Luckily in this instance the police arrested the carjackers [3]. Additionally, the Daily Guide reported that 11 carjackers were caught and arrested by policemen. Unfortunately, the policemen were unable to retrieve all the vehicles stolen by these carjackers and as such justice was not completely served [4].

From all these cases presented it is obvious that although the police can solve this problem of carjacking to an extent, there are still measures that can be taken to reduce its occurrence. As such, to solve this issue of carjacking, this project aims to fuse a mechanical

design with a computerized control mechanism. With the aid of a rack and pinion Ackermann steering system controlled with a mobile application , this project would help solve the issue of carjacking. The combination of these two features would provide an anti-car theft mechanism. However, the author of this paper would focus on the design of a rack and pinion Ackermann steering system which would accommodate the use of a remote control, while Mr. Samuel Bunyan would focus on the creation of a mobile application which would have a remote-control feature and a GPS tracking feature. More information on this project topic are detailed in the subsequent sections.

1.2 Problem definition

The problem as discussed in section 1.1, is the issue of carjacking which currently exists in Ghana. For the purposes of this capstone project, the aim is to design a rack and pinion Ackermann steering system which can be controlled with a mobile app. To effectively incorporate the use of the mobile app, a rear-wheel control mechanism would also be devised.

1.3 Scope of the project

1.3.1 Target market

The target market for this project are all vehicle owners based in Ghana.

1.3.2 Objectives

This project seeks to achieve certain objectives, of which smaller targets and goals are a means to an end. In that vein, this project hopes to firstly achieve the following:

A. Design of the following features in Solidworks:

- Front-wheel rack and pinion Ackermann steering system
- Rear-wheel drive control mechanism

B. Addition of unique features

- Remote Control: This feature is to properly fuse modern day technology into the standard toy make-up, we found it essential to include a remote control to make use of technological advancements to aid in the construction of this project. As such, with the use of a remote control, the vehicle that would be used to test if the steering system can be automatically controlled.
- GPS tracking module: This feature allows the user to know his/her vehicle's current location. It is a very important tool because whenever a vehicle gets missing, this feature allows it to be tracked down easily.

C. Manual testing of steering system

With the use of computer aided design (CAD) software such as Solidworks, a model of this project would be designed and tested to give a further idea of the quality of the project. The test to be carried out would be a static test to generate the safety factor of the design as well as the contact stresses and deformations present in the structure produced.

D. Implementation of rear-wheel drive control mechanism

To install the unique features as stated above (remote control and GPS tracking), a rear-wheel drive control mechanism is implemented. This system provides a holder for the DC motors used for the remote-control system, and provides a medium through which the DC motors could be used to control the rear wheels.

1.3.3 Expectations and acceptance

At the end of this project, it is expected that all the objectives mentioned in section 1.3.2 would be completed and thoroughly iterated to further improve the final product. The acceptance criteria for this project is simply a project that meets every single objective as stated in the preceding subsection. Any result that falls short of this would be regarded as a failure.

1.3.4 Project constraints

There are two constraints this project currently faces. These are as follows:

- Time constraint: To effectively produce this project, ideally two to three months should be allocated to only production. Unfortunately, only a month was allocated to production as there was a change to the design and materials used for the project.
- Availability of resources: Since not all the required resources for this project are readily available in Ghana, this proved to be a constraint. As such alternatives were procured to meet the project's objectives.

1.4 Motivation of project topic

Theft appears in many shapes or forms and unfortunately is a recurring problem in Ghana. Although there are law enforcers which enable the prevention and the control of these acts, there are still limitations in their methods; they lack technological traits. Thus, this project seeks to solve a type of theft; carjacking. By constructing an anti-car theft mechanism, car owners can easily prevent their cars from being stolen. Also, law enforcers in Ghana would find

it easier to track carjackers and stolen cars, thus improving their methods in eradicating this issue in Ghana.

1.5 Literature review

In this section, research was conducted to find out the status quo and the origins of toy cars and steering systems in general.

Firstly, from research, it was found that the first ever toy car was invented in 1952 by Jack Odell. At the time he invented it, he referred to this toy car as a steamroller. Quite surprisingly, this steamroller was purposely invented to prevent his daughter from playing with spiders. The first ever design was made of brass and was painted shiny red and green [5]. As the years have gone by, additional features have been added to toy cars. Some of these features include steering systems, which is the focus of this project. The subsequent sections will focus on a review of this feature.

1.5.1 Overview of steering systems

Based on existing literature, the following information about steering systems was found. These will be detailed further in the document. Firstly a steering system is a system that allows the motion of wheels from left to right by allowing the transmission of the steering wheel down the steering shaft [6]. There are two basic steering systems which are the rack and pinion and the recirculating ball bearings [7]. The rack and pinion steering system seemed the most viable due to the following advantages:

- Its small size - it can be used for a toy car [8].
- It is lighter than the other steering systems mentioned [8].

- It is easier to repair [8].

1.5.2 Functionality of a rack and pinion

Currently, in most vehicles, the most commonly used steering system is the rack and pinion steering system. This system involves the conversion of the rotational motion of the steering wheel into linear motion which in turn moves the wheels from left to right. Furthermore, it also involves the use of a circular gear also known as the steering pinion which locks teeth on a bar known as the rack [6].

1.5.3 Construction of a rack and pinion

To construct this steering system, a rack gear is contained or protected in a tubular casing. The casing is then maintained on the frame near its ends, with the ends of the rack being attached to the track rod with the help of a ball and socket joint. Furthermore, the pinion shaft is transmitted in the stark bearings housed in casing and the pinion is interlocked with the rack and clearance is attuned with a fine-tuning screw [9].

1.5.4 Practical applications of the rack and pinion steering system

From research, the following are practical applications of the rack and pinion steering system. They are:

- **Automatic car park system:** In this application, the rack and pinion makes up a conveyor mechanism for an automatic car park system. This steering system is preferred due to its compactness and lightness. With the aid of a stepper motor connected to the pinion gear, the movement of the conveyor system is controlled, as it enables it to be driven back and forth[10].

- **Quadbike:** It was mentioned that the steering geometry of a quadbike is commonly made up of a plate system. However, with the use of rack and pinion steering system, the stability of its steering system is improved [11].

1.5.5 Limitations of the rack and pinion steering system

Although the rack and pinion steering system is widely used, it still possesses a few risks whenever implemented. These are as follows:

- **Leakage:** Due to its simplicity, there is a larger strain on individual parts of the rack and pinion system and as such, the wear produced can cause leakages due to cracking of the components [12].
- **Low durability:** Due to its low strength, it easily weakens during operation on rough terrain [12].
- **Vibration:** Due to its construction being comprised of very few components, it is unable to withstand very high disturbances on rough terrain. As such more road feel is generated thus, noise and vibration can be transmitted to the drivers and passengers in the vehicle [12].

1.6 Proposed chapter outline

This project requires thorough intensive and extensive research and as such, to simplify this project, the subsequent chapters are outlined as follows:

- **Chapter 2: Design**

This section focuses on the design procedure required to achieve the project's objectives.

- Chapter 3: Methodology

This section provides a thorough description of the processes undertaken when fulfilling the projects objectives.

- Chapter 4: Results

This section focuses on the results gathered from the tests carried out and documented in the Methodology section.

- Chapter 5: Conclusion

This section discusses the outcomes obtained from the Results section, the limitations encountered throughout the project, and lastly, steps to be taken to improve this project in the future.

2 CHAPTER 2: DESIGN

2.1 Overview

In this chapter, all aspects of the design for this project would be thoroughly discussed.

The outline for this chapter is as follows:

- Design objective
- Review of existing designs
- Design iterations
- Design decision
- Material selection
- Design parameters
- Mathematical modeling

2.2 Design objective

Considering the types of steering systems, as discussed in Chapter 1, the rack and pinion steering system seemed the most viable. Currently, the rack and pinion system is one of the most commonly used steering systems due it being lightweight [9], and as such it made it more reasonable to design a steering system along its concept. The rack and pinion system is a rather simple system and is comprised of the following components:

- Rack and pinion gears
- Steering Shaft
- Steering Wheel
- Tie-rod Joints
- Wheels and Steering arm

The main idea of this design is to simply create a steering system with the components mentioned above to prevent carjacking. With the aid of a remote-control mechanism and a GPS tracking module constructed by my partner Samuel Bunyan, the steering system would be able to fulfill the objective of preventing carjacking. The materials that would be used for this design would be metals. The type of metal used would be decided in Section 2.6, which focuses on material selection.

Furthermore, to improve upon this design idea, it is essential to understand other existing designs that have been constructed. Section 2.3 focuses on a review of existing designs, which details the advantages and disadvantages of each design based on some certain criteria.

2.3 Review of existing designs

In this section, the following designs of rack and pinion steering systems would be reviewed:

- A rack and pinion steering system for a four-wheel drive car
- A steering gear design and a shaft for a rack and pinion steering system
- A steering mechanism for toy cars using a rack and pinion gear

These would be reviewed based on the following criteria: durability, aesthetics and complexity. Furthermore, the main purpose of this review is to aid in the completion of this project's design objective; it will assist in generating more ideas needed to be implemented in the design of the rack and pinion steering system for this project.

2.3.1 Rack and pinion steering system for a four-wheel drive car

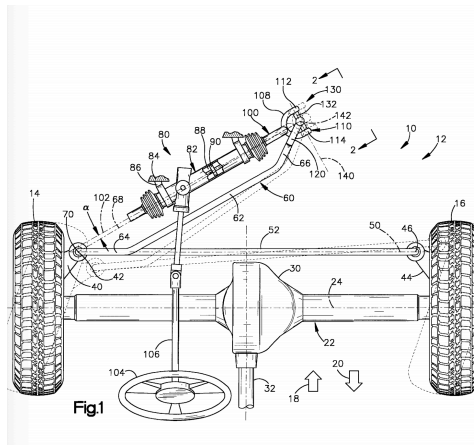


FIGURE 2-1: RACK AND PINION STEERING SYSTEM FOR A FOUR-WHEEL DRIVE CAR

SOURCE: ADAPTED FROM [13]

The design as shown above is for a rack and pinion steering system for a four-wheel drive car. In this design, a pinion steering gear is coupled with an elongated rack with a longitudinal axis and a live axle. The rack designed is positioned in such a way as to move axially as the steering wheel is rotated. Additionally, a link bar is connected between one end of the rack and the steering arms to transmit force to the steering arms. Lastly, the link bar has a kinematic axis which is parallel to the rack's axis. [13]

Upon a meticulous review of this design, it seems to fit all but one of the criteria mentioned in section 2.2; complexity. This is because the sketch seems over defined and as such it is a bit difficult to understand. In addition to that, the parts designed in the sketch were not labelled with words, but with numbers. This was a good strategy by the designer, however there were several numbers on the design which made it difficult to follow. The design is complex.

2.3.2 A steering gear design and a shaft for a rack and pinion steering system

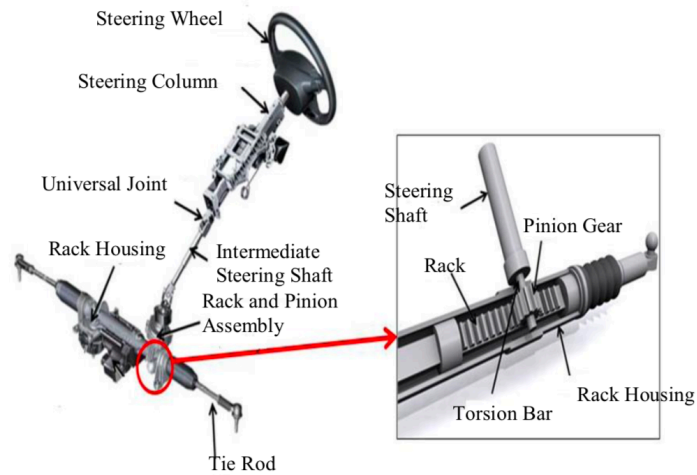


FIGURE 2-2: STEERING GEAR DESIGN AND A SHAFT FOR A RACK AND PINION STEERING SYSTEM

SOURCE: ADAPTED FROM [14]

For this design, it was stated that the designers wanted to make the design as simple as possible. From my point of view, it seems very simple and precise, and at first glance its mechanism looks very understandable. The design made use of a universal joint to link the rack housing to the steering column. For the pinion, the designers made use of only 6 teeth on the pinion and 28 teeth on the rack. The reason behind the number of teeth was to reduce the complexity of the design. Additionally, this design was tested and it showed that the minimum and maximum factor of safety were 3.8631 and 15 respectively, which shows that the design is durable. [14]. Lastly, the design as shown above looked appealing and well-constructed, thus it meets all the criteria stated in section 2.2.

2.3.3 A steering mechanism for toy cars using rack and pinion gears

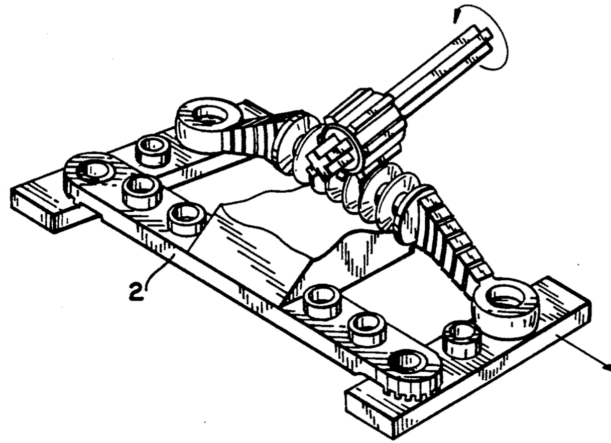


FIGURE 2-3: A STEERING MECHANISM FOR TOYS USING RACK AND PINION GEARS

SOURCE: ADAPTED FROM [15]

The design as shown above, is a rack and pinion steering system for a toy vehicle. The designers made use of a compact structure, comprised of a rack, pinion gear, steering rod and a bushing. In this design, the bushing was coupled with the steering gear housing which contained both the rack and the pinion. Additionally, there are two parallel pipes which extend diagonally towards the longitudinal axis of the rack. [15]

After a careful review of this design it is evident that there is not enough information to conclude on the durability of the design. However, the design looked aesthetically pleasing but slightly complex due to the connection components as seen in Fig. 2-3.

All the designs reviewed above shared something in common, and that was an innovative approach either through complexity, compactness or even pure simplicity. Moving on, the design for this project would aim to satisfy an element of uniqueness.

2.4 Design Iterations

In constructing the design of this project, three designs were implemented. The difference between these three designs was the number of teeth. These designs can be seen in Fig. 2-4, below.



FIGURE 2-4: 10 TEETH (LEFT), 20 TEETH (CENTER) AND 30 TEETH (RIGHT), FOR THE PINION GEAR

After careful consideration, a pinion gear with 30 teeth was selected simply due to its aesthetics and strength.

2.5 Design Decision

In this section, a decision on which steering system is most appropriate for this project is made with the aid of a Pugh Matrix. Considering that in Chapter 1 the rack and pinion steering system was chosen over the recirculating ball bearings steering system, the Pugh matrix as shown below in Table 2-1, further explains why the rack and pinion steering system is a much better fit for this project.

TABLE 2-1: PUGH MATRIX

| Criteria | Recirculating ball bearings steering system | Rack and pinion steering system |
|------------------|---|---------------------------------|
| Cost | 0 | 0 |
| Complexity | – | + |
| Aesthetics | + | + |
| Overall Size | - | + |
| Durability | + | - |
| Sum +’s | 2 | 3 |
| Sum 0’s | 1 | 1 |
| Sum – ‘s | 2 | 1 |
| Net Score | 0 | 2 |

From the Pugh Matrix as shown above, it is evident that based on the criteria above the rack and pinion steering system would be more suitable for the project.

2.6 Material Selection

Due to the availability of materials in my location, the materials to be selected for this project were limited to just two; poly lactic acid (PLA) and aluminum alloys. With the help of the *CES Edupack* software, the metal which was most suitable for this project was selected. This selection was based on density and yield strength. Figure 2-5 below shows the results from the study carried out using the software.

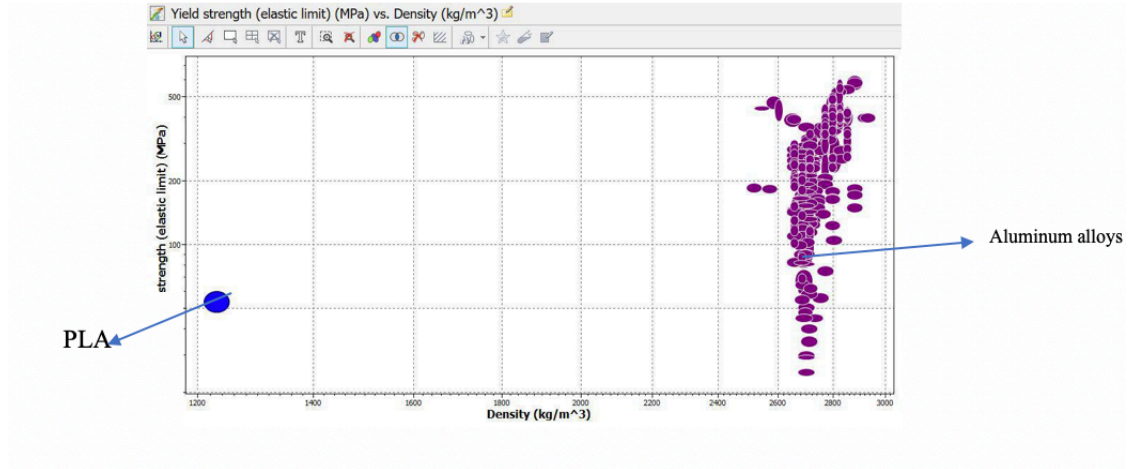


FIGURE 2-5: A PLOT OF DENSITY AGAINST YIELD STRENGTH FOR MATERIAL SELECTION

From the results above it is seen that PLA will be the material best suitable for the design. Even though some alloys of aluminum and PLA both had similar ranges in the yield strength, the density was the deciding factor. This is due to the fact that in section 2.2, the steering design had to be lightweight. From Figure 2-5 as shown above, it is observed that the density of PLA was lower than 1400kg/m^3 whereas the density of the aluminum alloys was between 2400kg/m^3 and 3000kg/m^3 . Thus, this makes PLA lighter.

2.7 Design Parameters

To construct a rack and pinion steering system, the following quantities were measured to generate a CAD model on Solidworks. These quantities include, the diameter, length, number of teeth and the pressure angle of both the rack and pinion gears. Afterwards, these measurements were recorded and are shown in Table 2-2 below.

TABLE 2-2: DESIGN PARAMETERS FOR THE RACK AND PINION GEARS

| Quantity | Value |
|---------------------------|--------|
| Width of rack | 2 cm |
| Length of rack | 15 cm |
| Number of teeth on rack | 24 |
| Number of teeth on pinion | 30 |
| Diameter of pinion | 6.5 cm |
| Pressure Angle | 20° |

From the parameters above, the generated CAD model of both the rack and the pinion gears can be seen below in Fig. 2-6.

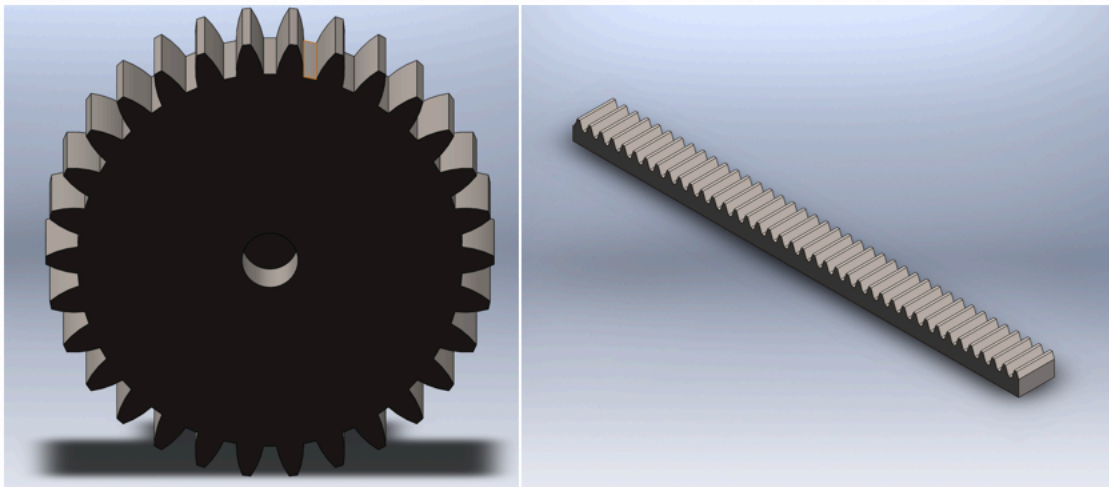


FIGURE 2-6: PINION GEAR (LEFT) & RACK GEAR (RIGHT)

From Fig. 2-6 as shown above, certain quantities were measured in Solidworks and recorded in Table 2-3 below. The module of the rack and pinion was calculated from the ratio of the diameter of the pinion to the number of teeth on the pinion. Lastly, the gear ratio was calculated

and recorded from the ratio of the number of teeth on the rack gear to number of teeth on the pinion gear.

TABLE 2-3: GENERATED PARAMETERS FROM SOLIDWORKS

| Quantity | Value |
|---------------------------|----------|
| Pitch | 0.6 cm |
| Tooth thickness | 0.4 cm |
| Module of rack and pinion | 0.217 cm |
| Addendum | 0.2 cm |
| Dedendum | 0.3 cm |
| Tooth depth | 0.5 cm |
| Gear ratio | 0.8 |

The gear ratio of 0.8 as shown in Table 2-3 shows that for a 100% rotation of the pinion gear, the rack gear is driven by 80% linearly.

2.8 Mathematical Modelling

To complete the production of the steering system, several steering gear mechanisms were explored. These include Parallel Steering, Ackermann Steering and Anti-Ackermann Steering. Weighing the advantages and disadvantages of all these three with respect to this project's design objective, the Ackermann Steering system was most suitable. This is because it prevents tires from skidding centrifugally when the wheels of a vehicle trail about a curve while taking a turn [16].

This steering mechanism was devised by Rudolph Ackermann in the early 1800's [17].

In this steering mechanism he stated that a geometric arrangement of linkages in the steering of a car is developed to turn the inner and outer wheels at suitable angles [18]. In addition, the Ackermann geometry involves the use of components such as a rack, pinion and tie rods [19]. Pertaining to this project, the Ackermann steering system would be used to govern the mathematical model of the steering system designed. Additional quantities were calculated from the model generated. These include the following: inner and outer turning circle radius, steering ratio and rack travel, steering wheel torque and lastly the torque on the pinion gear.

2.8.1 Ackermann steering model

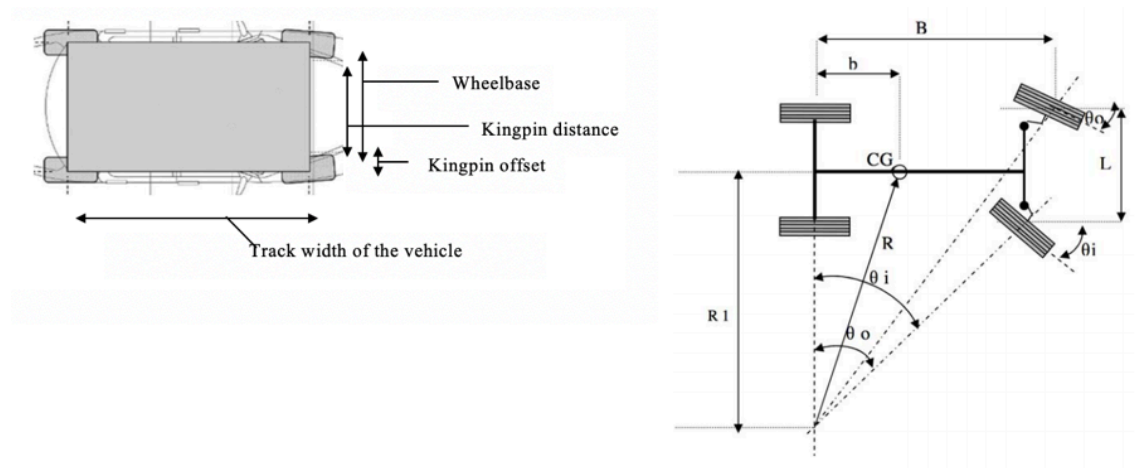


FIGURE 2-7: 2D IMAGE SHOWING ACKERMANN STEERING GEOMETRY

SOURCE: ADAPTED FROM [16]

From Fig 2-7 as shown above:

θ_o = outer wheel angle

θ_i = inner wheel angle

B = track width of the vehicle

L = base of the wheel

b = distance from the rear axle to the center of mass

TABLE 2-4: DESIGN PARAMETERS RETRIEVED FROM KIDS GO-KART AFTER THE IMPLEMENTATION OF THE RACK AND PINION STEERING SYSTEM

| Quantity | Value |
|---|------------|
| Base of the wheel | 0.61 m |
| Track width of the wheel | 0.66 m |
| Distance from the rear axle to the center of mass | 0.22 m |
| Kingpin distance | 0.27 m |
| Distance from the center of mass to the point where the outer and inner angles converge | 0.70 m |
| Width of the tire | 0.085 m |
| Kingpin offset | 0.075 m |
| Weight of the front axle | 0.08982 kg |
| Diameter of pinion | 0.065 m |

Using Ackermann steering geometry:

EQUATION 2-1: ACKERMANN STEERING GEOMETRY

$$\cot\theta_o - \cot\theta_i = \frac{B}{L} \quad [16] \quad (2.1)$$

The equation as shown above is used to generate the angles at which the front wheels of the vehicle would turn.

To find θ_i and θ_o , referring to Fig. 2-7, using Pythagoras theorem,

$$R = \sqrt{R1^2 + B^2}$$

$$R^2 = R1^2 + B^2$$

$$R1 = \sqrt{R^2 - B^2}$$

$$R1 = \sqrt{0.70^2 - 0.66^2}$$

$$R1 = 0.361 \text{ m}$$

$$\text{Also, } R1 = \frac{B}{\tan(\theta_i)}$$

$$0.361 = \frac{0.66}{\tan(\theta_i)}$$

$$\theta_i = 61.3^\circ$$

$$\cot\theta_o - \cot(61.3^\circ) = \frac{0.66}{0.61}$$

$$\theta_o = 31.5^\circ$$

Thus, the outer and inner wheel angles are 31.5° and 61.3° respectively.

Furthermore, to calculate the Ackermann angle:

EQUATION 2-2: ACKERMANN ANGLE

$$\alpha = \tan^{-1}((K_d/2)/L) \quad [16] \quad (2.2)$$

Where K_d = kingpin distance

$$\alpha = \tan^{-1}((0.27 \text{ m} / 2) / 0.61 \text{ m}) = 12.5^\circ$$

The Ackermann angle is the angle created when the left front wheel reaches its maximum inner turning angle while the right front wheel reaches its maximum outer turning angle or vice versa.

2.8.2 Turning circle radius

The turning circle radius is the diameter of the smallest circle after every turn [16]. This calculation gives an indication of the space needed to turn a vehicle [20].

Solving for the inner turning circle radius:

EQUATION 2-3: INNER TURNING CIRCLE RADIUS

$$R_{if} = \frac{l}{\sin \theta} + \frac{B - K_d}{2} \quad [16] \quad (2.3a)$$

$$R_{if} = \frac{0.61 \text{ m}}{\sin 30} + \frac{0.66 \text{ m} - 0.27 \text{ m}}{2} = 1.415 \text{ m}$$

This shows that after every turn, the diameter of the smallest circle is 1.415 m.

Solving for the outer turning circle radius:

EQUATION 2-4: OUTER TURNING CIRCLE RADIUS

$$R_{of} = \frac{l}{\sin \phi} + \frac{B - K_d}{2} \quad [16] \quad (2.4b)$$

$$R_{of} = \frac{0.61 \text{ m}}{\sin 19.56} + \frac{0.66 \text{ m} - 0.27 \text{ m}}{2} = 2.017 \text{ m}$$

This shows that after every turn, the outer diameter of the smallest circle created when the vehicle turns would be 2.017 m.

2.8.3 Steering ratio and rack travel

The steering ratio is simply the ratio between the maximum steering wheel movement and the maximum turn of a steering wheel [16]. Therefore, for a maximum turn to be 25° and for the maximum steering wheel movement to be 180°, the steering ratio is as follows:

$$\text{Steering ratio} = \frac{180}{25} = 7.2$$

In addition, the rack travel is the horizontal movement by the rack gear as the pinion gear completes a rotation [21]. It is also the ratio of the area of the steering wheel and the steering ratio.

The steering wheel travel for one rotation = $2\pi r = 2\pi(8.25\text{cm}) = 51.8\text{cm}$

Thus, the rack travel = $\frac{51.8\text{cm}}{7.2} = 7.2\text{cm}$. This means that the rack will move by 7.2 cm after a rotation by the pinion gear.

2.8.4 Kingpin torque

To find the torque generated on the Kingpin, the following equation was used:

EQUATION 2-5: KINGPIN TORQUE

$$T_k = F_w u \sqrt{\frac{w_t^2}{8} + e^2} \quad [16] \quad (2.5)$$

where T_k = Torque produced by the Kingpin

F_w = Weight of the front axle

u = coefficient of friction

w_t = width of the tyre

$e = \text{Kingpin offset}$

The design for this steering system was intended for smooth horizontal terrains and as such the coefficient of friction, μ is 0.6 [22].

The kingpin offset is the distance between the center of the wheel and the steering arm.

$$\text{Thus, } T_k = 0.08982(0.6)\sqrt{\frac{0.085^2}{8} + 0.075^2}$$

$$T_k = 4.35 \times 10^{-3} \text{ N.m}$$

2.8.5 Steering wheel torque

To find the torque generated on the steering wheel, the following equation will be used:

EQUATION 2-6: STEERING WHEEL TORQUE

$$\text{Steering wheel torque} = \text{Steering ratio} \times \text{Kingpin torque} \quad (2.6)$$

Thus the steering wheel torque of the steering system is as follows:

$$\text{Steering wheel torque} = 7.2 \times 4.35 \times 10^{-3} \text{ N.m}$$

$$\text{Steering wheel torque} = 0.03132 \text{ N.m}$$

2.8.6 Torque on pinion gear

To find the torque generated on the pinion gear, the following equation will be used:

EQUATION 2-7: TORQUE ON PINION GEAR

$$\text{Torque on pinion gear} = \frac{\text{Kingpin torque} \times \text{Gear ratio}}{\text{Diameter of pinion}} [16] \quad (2.7)$$

$$\text{Torque on pinion gear} = \frac{4.35 \times 10^{-3} \text{ N.m} \times 0.8}{0.065 \text{ m}} = 0.054 \text{ N.m}$$

Thus, this value of torque on the pinion gear will be used when carrying out the simulation study in Solidworks (see Appendix A-4), which would be discussed in Chapter 3 and Chapter 4.

3 CHAPTER 3: METHODOLOGY

In this section, the procedures carried out in order to meet the objectives defined in section 1.3.2, are thoroughly described. The outline for this chapter is as follows:

- Modelling of the rack and pinion steering system in Solidworks.
- Implementation of rack and pinion steering system on a go-kart.
- Implementation of rear-wheel drive control mechanism.

3.1 Modeling of rack and pinion steering system in Solidworks

Firstly, the parameters as discussed in section 2.7 for the rack gear and pinion gear were defined and thus, each gear was generated. In addition, the rack gear was designed in a tubular shape, to add an element of innovation. The other components such as the steering shaft, steering wheel, tie-rod Joints, steering arm and tyres were all designed and coupled in Solidworks. This can be seen in Fig. 3-1 as shown below.

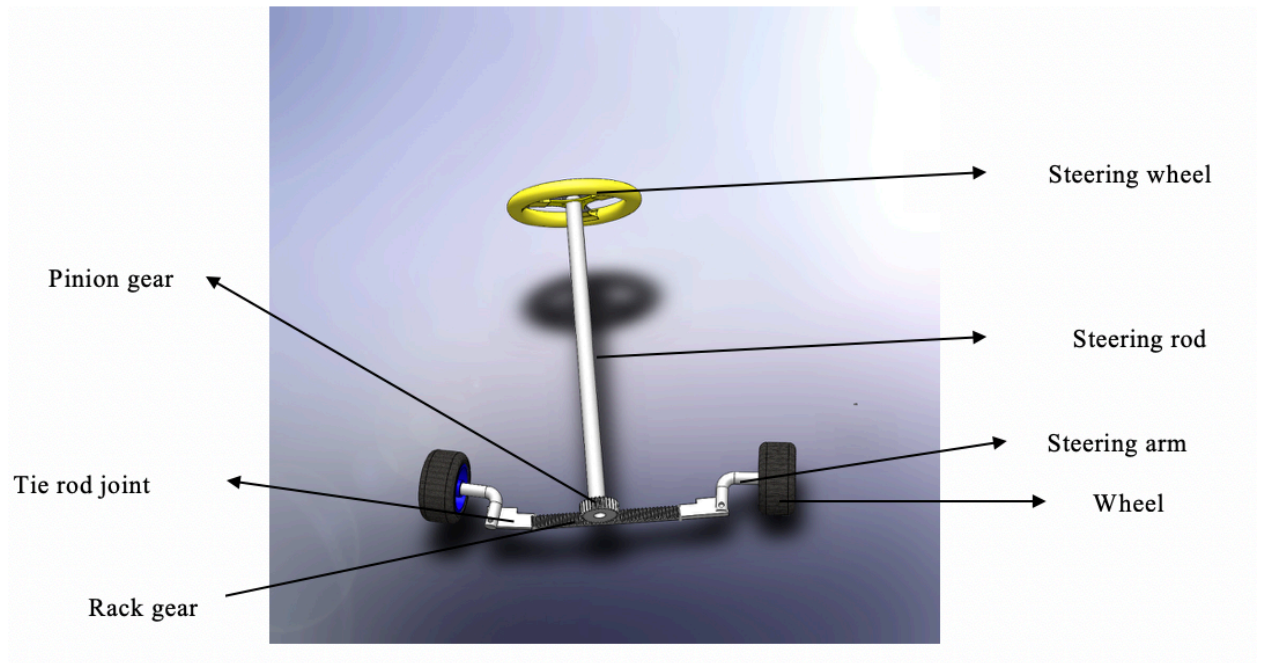


FIGURE 3-1: STEERING ASSEMBLY IN SOLIDWORKS

To validate the safety of the rack and pinion steering system designed, a static test was conducted on assembly of the rack and the pinion. Section 3.1.1 details each test conducted.

3.1.1 Simulation in Solidworks

In running a simulation in Solidworks, a static test was conducted on an assembly of the rack and pinion. In this test the value calculated in section 2.8.6 was applied onto the pinion gear, see Appendix A-4. This test was used to generate the following: the factor of safety test, total deformation test and a von Mises stress test.

- Factor of Safety

The factor of safety is conducted to observe the maximum stress that an individual part or an assembly can endure in comparison with the maximum stress that an individual part or an assembly will be exposed to in its process [23].

- Total Deformation

In this test, loads are subjected at specific points on an individual part to observe how much load the individual part can withstand.

- von Mises stress

In the von Mises stress test, an equation records each shear and principal stress value, and produces a single von Mises stress value. In order to produce a structure that is not failing, the single von Mises stress value generated must be less than the yield strength of the material [24].

3.2 Implementation of rack and pinion steering system on a go-kart

To implement the rack and pinion steering system on the go kart, the following steps were carried out:

- The rack gear and the pinion gear designed in the Solidworks model were 3D printed with the use of PLA as its material. These prints were carried out using an *Ultimaker* 3D printer as seen in Fig. 3-1 below.

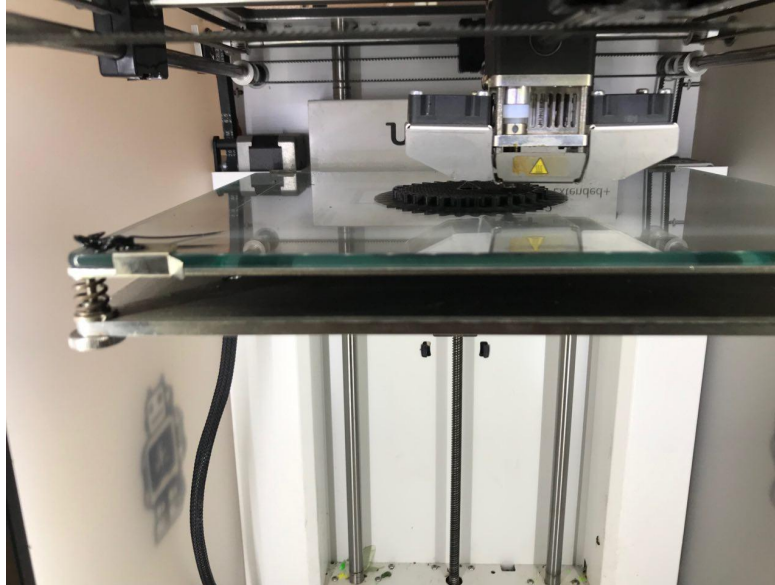


FIGURE 3-2: PINION GEAR IN A 3D-PRINTER

The 3D printed rack and pinion gears can be seen in Fig. 3-2 below.



FIGURE 3-3: 3D-PRINTED RACK AND PINION GEARS

- The existing steering system on the go-kart was disconnected. The only components maintained were the steering arm connected to the tyres and the chassis, as they were also key components for the rack and pinion steering system. This can be seen in Fig. 3-3 below.



FIGURE 3-4: MODIFIED GO-KART

- The rack gear was then attached to tie rod joints, to provide a connection to the tyres.
- An aluminum rod was measured and this made up the steering shaft of the system.
- The steering shaft was connected through holes drilled in the chassis of the go kart, with a steering wheel connected to it at its apex.
- A hole was drilled within the pinion gear, to allow the connection of an aluminum rod (steering shaft) to the steering wheel.
- The pinion gear was then attached freely to the rack gear.
- The structure was tweaked until the steering was efficient.

Figure 3-2 below shows the implementation of the rack and pinion system on the go-kart.



FIGURE 3-5: IMPLEMENTED RACK AND PINION STEERING DESIGN ON GO-KART

3.2.1 Manual testing of steering system

In this section the testing of the steering system was done by verifying the Ackermann conditions calculated in Chapter 2. To verify the conditions, the angles were measured with the use of a protractor. The angles measured were the outer wheel angle, the inner wheel angle and the Ackermann angle.



FIGURE 3-6: MANUAL VERIFICATION OF ACKERMANN STEERING CONDITIONS

3.3 Implementation of rear-wheel drive control mechanism

To install the remote-control circuits on the rear wheels of the vehicle, a rear wheel drive mechanism was designed in Solidworks which can be seen in Fig. 3-7 below.

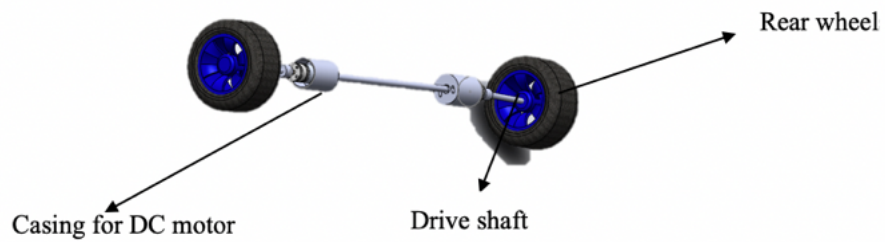


FIGURE 3-7: REAR-WHEEL DRIVE CONTROL MECHANISM

This design was made to aid in the construction of the rear-wheel drive control mechanism by providing a virtual representation of the system. This system was then assembled onto the go kart, which can be seen in Fig. 3-8 below.



FIGURE 3-8: REAR-WHEEL DRIVE CONTROL MECHANISM IMPLEMENTED ON KIDS GO-KART

4 CHAPTER 4: RESULTS AND DISCUSSION

In this section, the results from the tests discussed in Chapter 3 are recorded.

4.1 Results from Solidworks simulation

The results as shown in the tables below were recorded and published by Solidworks.

TABLE 4-1: MINIMUM AND MAXIMUM VON MISES STRESS RECORDED DURING THE STATIC TEST

| Name | Type | Min | Max |
|---------|-----------------------|--|--|
| Stress1 | VON: von Mises Stress | 9.146e-04 N/m ² Node: 2062 | 8.899e+05 N/m ² Node: 7017 |

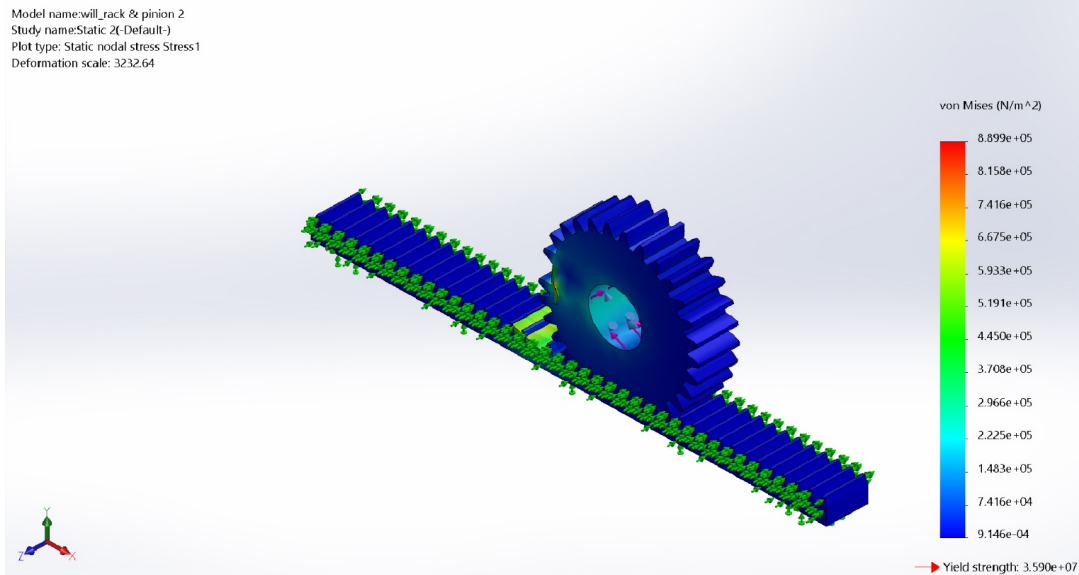


FIGURE 4-1: RANGE OF VALUES OF VON MISES STRESS UNDERGONE BY RACK AND PINION DURING STATIC TEST

TABLE 4-2: MINIMUM AND MAXIMUM RESULTANT DISPLACEMENT RECORDED DURING THE STATIC TEST

| Name | Type | Min | Max |
|---------------|------------------------------|--------------------------|----------------------------|
| Displacement1 | URES: Resultant Displacement | 0.000e+00 mm Node: 42 | 7.743e-03 mm Node: 5789 |

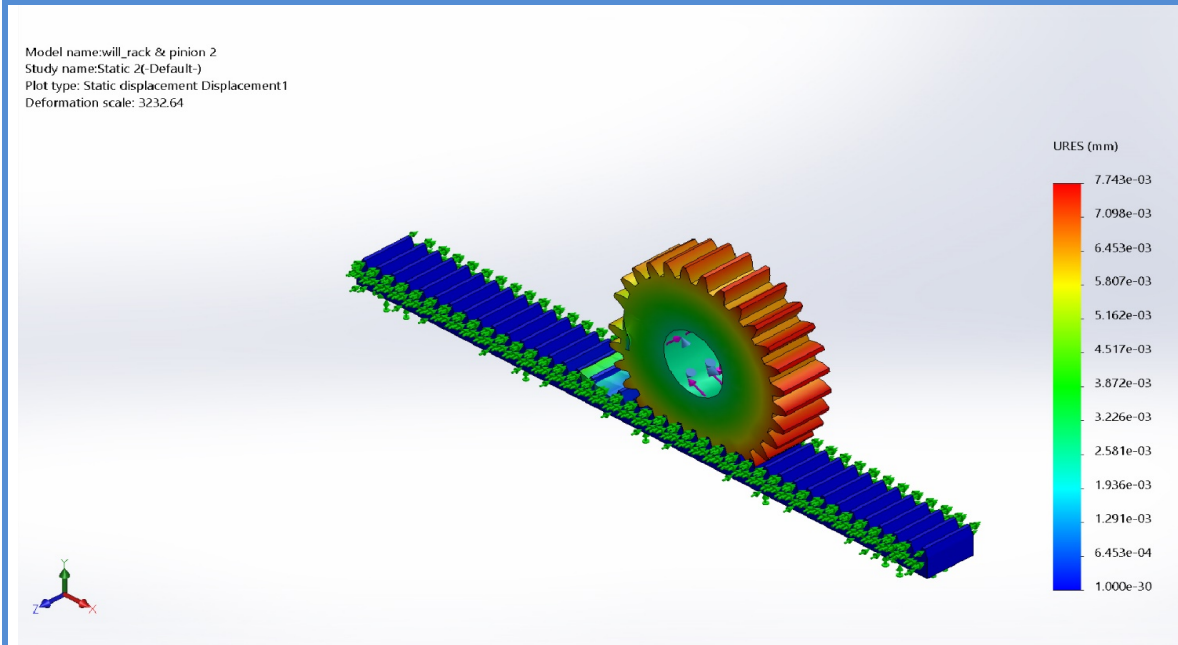


FIGURE 4-2: DISPLACEMENT UNDERGONE DURING STATIC TEST

TABLE 4-3: MINIMUM AND MAXIMUM EQUIVALENT STRAIN RECORDED DURING THE STATIC TEST

| Name | Type | Min | Max |
|---------|--------------------------|---------------|---------------|
| Strain1 | ESTRN: Equivalent Strain | 2.793e-13 | 3.035e-04 |
| | | Element: 2704 | Element: 3857 |

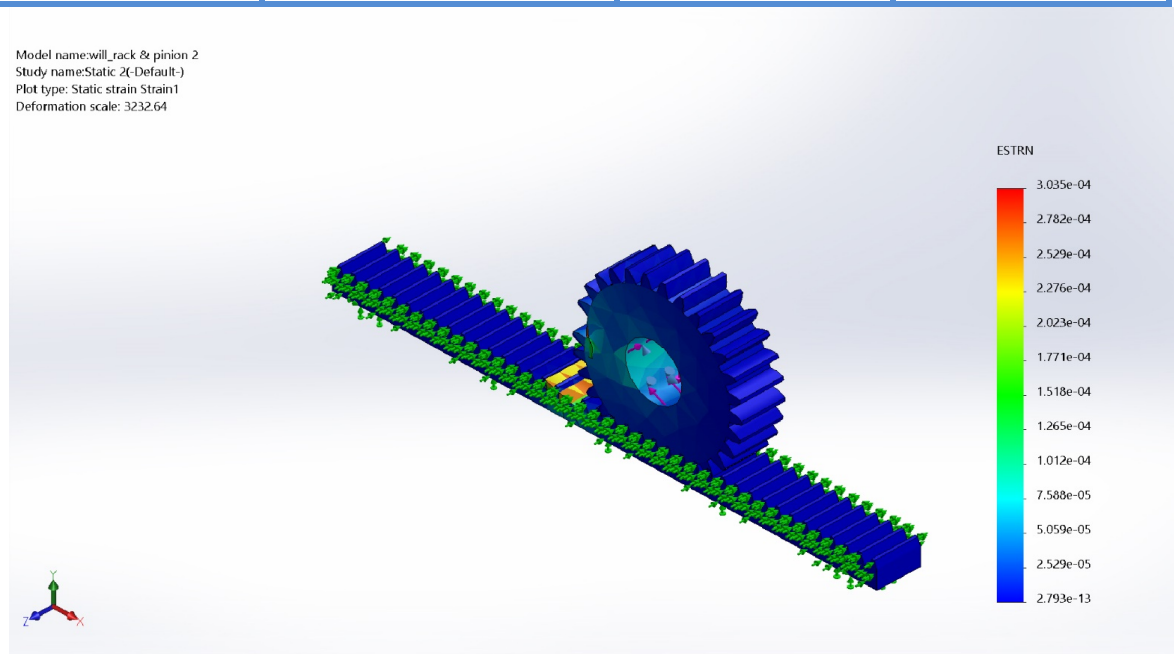


FIGURE 4-3: RANGE OF VALUES OF THE EQUIVALENT STRAIN BY RACK AND PINION DURING STATIC TEST

TABLE 4-4: MINIMUM AND MAXIMUM VALUES OF THE FACTOR OF SAFETY OF THE RACK AND PINION RECORDED DURING THE STATIC TEST

| Name | Type | Min | Max |
|-------------------|-----------|------------|------------|
| Factor of Safety1 | Automatic | 4.034e+01 | 3.925e+10 |
| | | Node: 7017 | Node: 2062 |

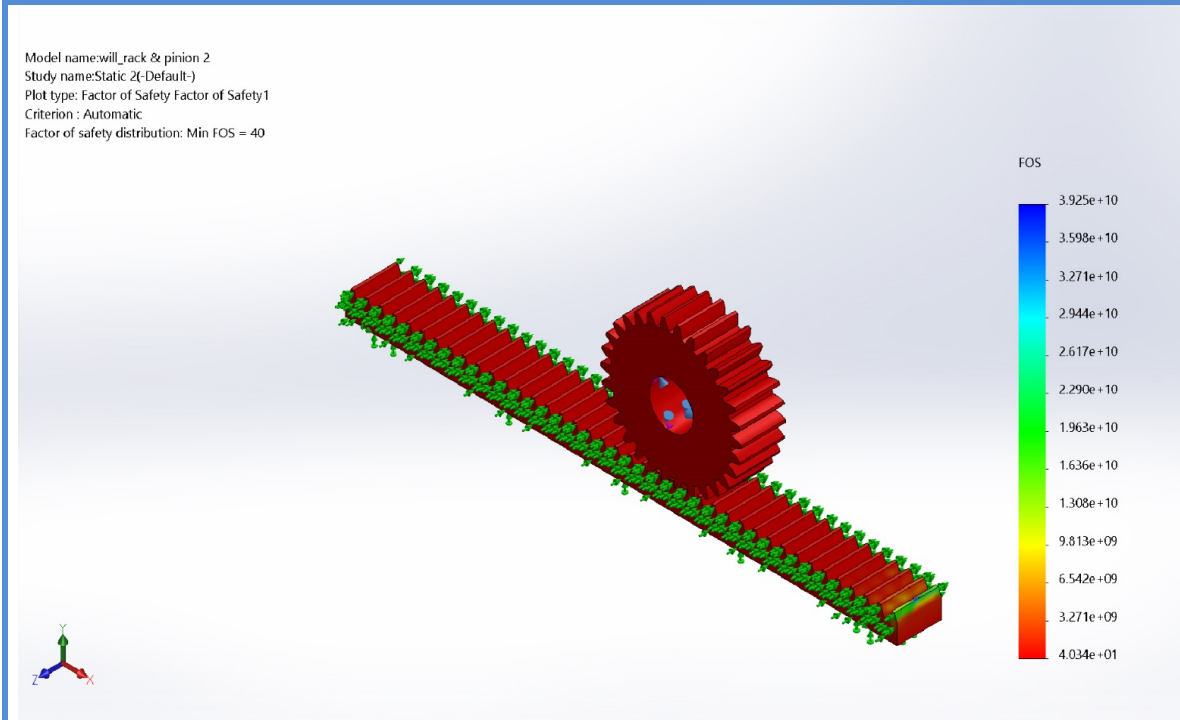


FIGURE 4-4: RANGE OF VALUES OF FACTOR OF SAFETY OF THE RACK AND PINION DURING STATIC TEST

TABLE 4-5: DEFORMED SHAPE OF RACK AND PINION

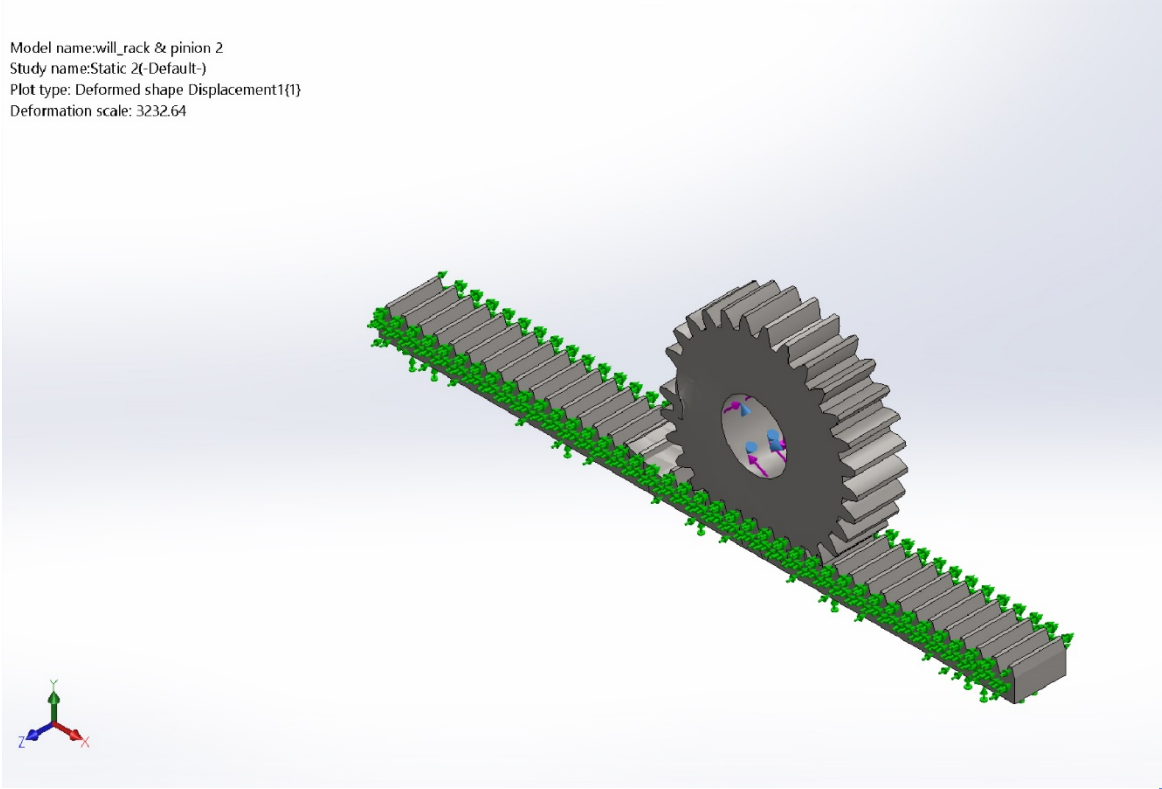
| Name | Type |
|---|----------------|
| Displacement1 | Deformed shape |
| <div><div>Model name:will Rack & pinion 2 Study name:Static 2(-Default-) Plot type: Deformed shape Displacement1{1} Deformation scale: 3232.64</div></div> | |

FIGURE 4-5:DEFORMED SHAPE OF RACK AND PINION DURING STATIC TEST

4.2 Results from testing of steering system

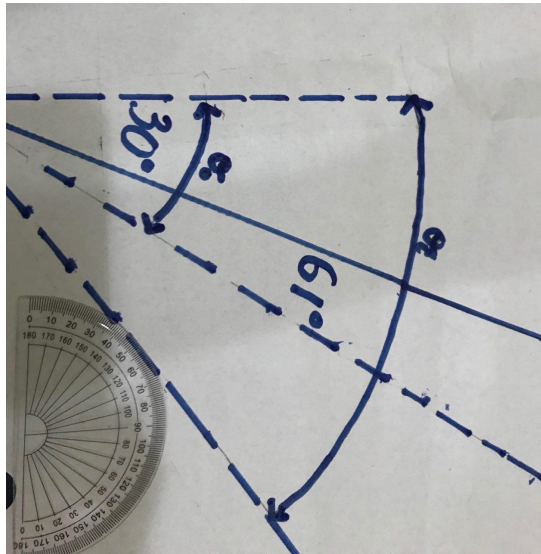


FIGURE 4-6: OUTER WHEEL ANGLE (30°) AND INNER WHEEL ANGLE (61°) OF STEERING SYSTEM

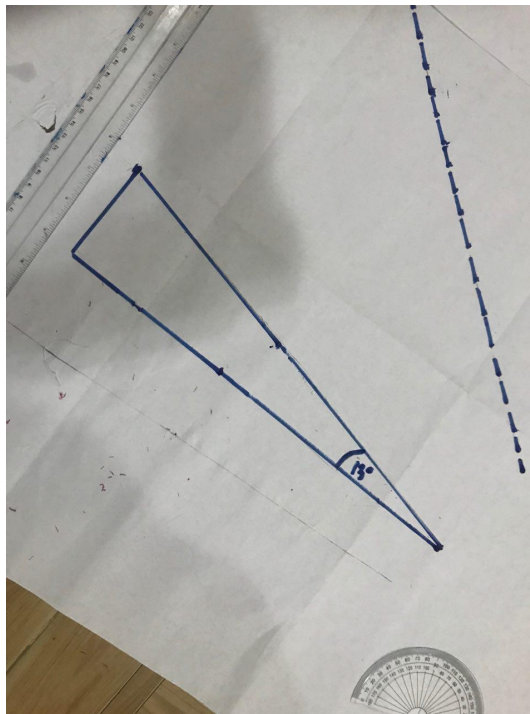


FIGURE 4-7: ACKERMANN ANGLE (13°) MEASURED WHEN VERIFYING ACKERMANN STEERING CONDITIONS

4.3 Discussion of results

The results recorded from the two tests undergone prove that the system designed is fully functional. Firstly, from the results recorded from Table 4-1, it is observed that the rack and pinion steering system, undergoes very minimal stresses when in contact. Also, from Table 4-2, it is shown that the rack and pinion steering system does not undergo any significant deformation during the study. In addition, for the factor of safety recorded in Table 4-3, the minimum and maximum values recorded were 4.034 and 3.925×10^{10} . This shows that the rack and pinion steering system is durable and will not fail as its value is above 1.

Lastly, from Fig. 4-1, it is seen that the steering system designed satisfy the Ackermann steering conditions calculated in Chapter 2, although, the values differ by very little margins. The difference in values could be due to inaccurate lines drawn and due to parallax error when reading from a protractor.

5 CHAPTER 5: CONCLUSION

Pertaining all the objectives outlined in Chapter 1, this project succeeded in meeting all of them. Both the front-wheel steering system and rear-wheel control drive were successfully implemented onto the kids go-kart. However, in doing so, there were a few obstacles that were encountered which almost hindered the completion of the project. These are detailed in section 5.1 below.

5.1 Limitations

This section details out the challenges encountered in this project. During this project, there were two major drawbacks faced and as such certain modifications were made.

- Unavailability of a functional milling machine for gear cutting

This proved to be a major stumbling block since this project relied solely on an accurate gear cutting machine. In the design of both the rack gear and the pinion gear, due to the lack of a milling machine for gear cutting, it was extremely difficult to cut out a gear with the dimensions specified in section 2.7. As such, these gears were 3D printed instead. Thus, the results generated in Chapter 4 were not a true reflection of what was built, since PLA was used instead of aluminum.

- Frequent 3D printing errors

During the printing of the pinion gear, the *Ultimaker* 3D printing machine encountered a few errors. The filament loaded into the machine had chipped off at certain ends which produced inaccurate tooth thicknesses for the pinion gear. Also, the filament frequently jammed, which caused prints to fail.

5.2 Future works

This section focuses on the aspects of this project that could be further improved. Some of these improvements are as follows:

- A. Addition of a steering wheel lock mechanism: This mechanism would ensure that the steering wheel does not rotate as the pinion gear is driven by a motor.
- B. Use of a milling machine for rack and pinion gears: This would ensure that metals can be a desirable material for gear production as all the gears would be cut accurately with the use of this equipment.

Appendix A

TABLE A- 1: STUDY PROPERTIES

| | |
|---|---|
| Study name | Static 2 |
| Analysis type | Static |
| Mesh type | Solid Mesh |
| Thermal Effect: | On |
| Thermal option | Include temperature loads |
| Zero strain temperature | 298 Kelvin |
| Include fluid pressure effects from SOLIDWORKS Flow Simulation | Off |
| Solver type | FFEPlus |
| Inplane Effect: | Off |
| Soft Spring: | Off |
| Inertial Relief: | Off |
| Incompatible bonding options | Automatic |
| Large displacement | Off |
| Compute free body forces | On |
| Friction | Off |
| Use Adaptive Method: | Off |
| Result folder | SOLIDWORKS document (C:\USERS\WILFRED AMOO-G\DOWNLOADS\RACK-AND-PINION-32.SNAPSHOT.3 (1)) |

TABLE A- 2: UNITS

| | |
|---------------------|------------------|
| Unit system: | SI (MKS) |
| Length/Displacement | mm |
| Temperature | Kelvin |
| Angular velocity | Rad/sec |
| Pressure/Stress | N/m ² |

TABLE A- 3: MATERIAL PROPERTIES

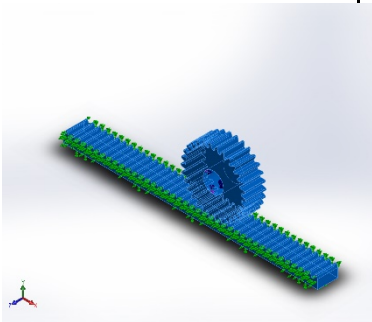
| Model Reference | Properties | Components |
|---|---|---|
|  | Name: Polylactic acid | SolidBody 1(TeethCuts)(rack spur rectangular_am-1), SolidBody 1(Bore)(spur gear_am-1) |
| | Model type: Linear Elastic Isotropic | |
| | Default failure criterion: Max von Mises Stress | |
| | Yield strength: 3.59e+07 N/m ² | |
| | Tensile strength: 2.64e+07 N/m ² | |
| | Elastic modulus: 2.3e+09 N/m ² | |
| | Poisson's ratio: 0.35 | |
| | Mass density: 1020 kg/m ³ | |
| Curve Data:N/A | | |

TABLE A- 4: LOADS

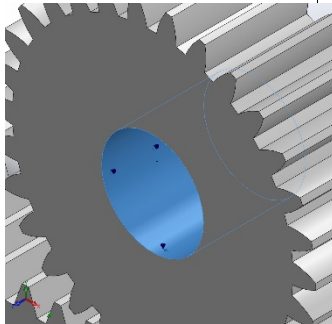
| Load name | Load Image | Load Details |
|-----------|---|--|
| Torque-1 |  | <p>Entities: 1 face(s)</p> <p>Type: Apply torque</p> <p>Value: 0.054 N.m</p> |

TABLE A- 5: CONNECTOR DEFINITIONS

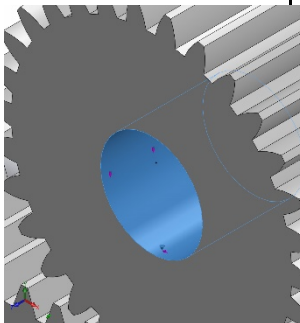
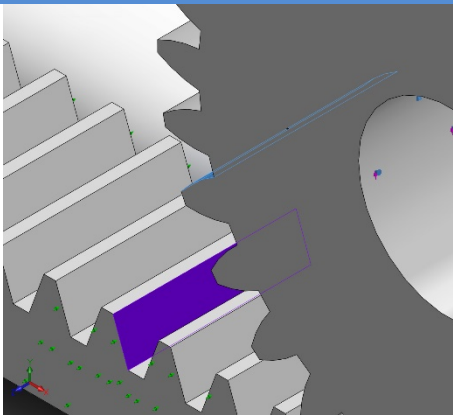
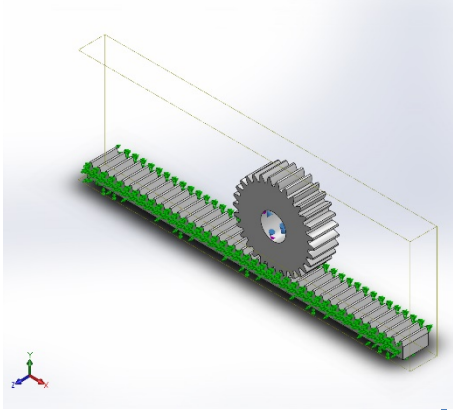
| Model Reference | Connector Details | | | |
|---|---|-------------|-------------|------------|
|  | <p>Entities: 1 face(s)</p> <p>Type: Bearing</p> | | | |
| Bearing Support-1 | | | | |
| Connector Forces | | | | |
| Type | X-Component | Y-Component | Z-Component | Resultant |
| Axial Force (N) | 8.0187e-19 | 1.3164e-18 | 0.025641 | -0.025641 |
| Shear Force (N) | -33.308 | -19.431 | 2.04e-15 | 38.562 |
| Bending moment (N.m) | -1.0576e-22 | -1.0591e-22 | 8.7453e-39 | 1.4974e-22 |

TABLE A- 6: CONTACT INFORMATION

| Contact | Contact Image | Contact Properties |
|----------------|---|---|
| Contact Set-1 |  | <p>Type: Bonded contact pair</p> <p>Entites: 2 face(s)</p> |
| Global Contact |  | <p>Type: Bonded</p> <p>Components: 1 component(s)</p> <p>Options: Compatible mesh</p> |

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